統計學

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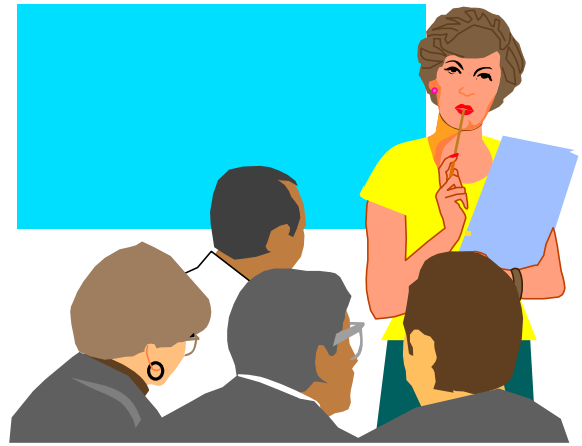
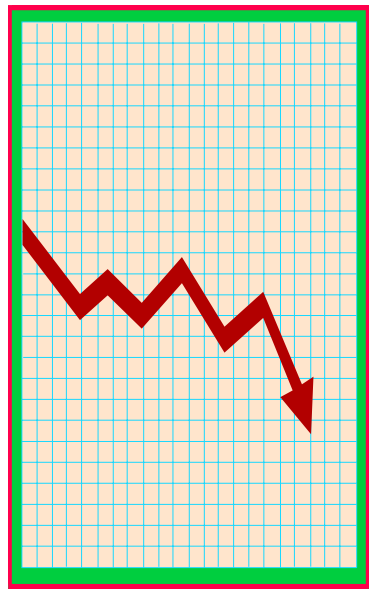


Chapter 18
Forecasting

- Time Series and Time Series Methods
- Components of a Time Series
- Smoothing Methods
- Trend Projection
- Trend and Seasonal Components
- Regression Analysis
- Qualitative Approaches to Forecasting



Time Series and Time Series Methods

- By reviewing historical data over time, we can better understand the pattern of past behavior of a variable and better predict the future behavior.
- A time series is a set of observations on a variable measured over successive points in time or over successive periods of time.
- The objective of time series methods is to discover a pattern in the historical data and then extrapolate the pattern into the future.
- The forecast is based solely on past values of the variable and/or past forecast errors.

The Components of a Time Series

- Trend Component
  - It represents a gradual shifting of a time series to relatively higher or lower values over time.
  - Trend is usually the result of changes in the population, demographics, technology, and/or consumer preferences.
- Cyclical Component
  - It represents any recurring sequence of points above and below the trend line lasting more than one year.
  - We assume that this component represents multiyear cyclical movements in the economy.

The Components of a Time Series

- Seasonal Component
  - It represents any repeating pattern, less than one year in duration, in the time series.
  - The pattern duration can be as short as an hour, or even less.
- Irregular Component
  - It is the "catch-all" factor that accounts for the deviation of the actual time series value from what we would expect based on the other components.
  - It is caused by the short-term, unanticipated, and nonrecurring factors that affect the time series.

Forecast Accuracy

- Mean Squared Error (MSE)
  - It is the average of the sum of all the squared forecast errors.
- Mean Absolute Deviation (MAD)
  - It is the average of the absolute values of all the forecast errors.

One major difference between MSE and MAD is that the MSE measure is influenced much more by large forecast errors than by small errors.

Using Smoothing Methods in Forecasting

- Moving Averages
  - We use the average of the most recent n data values in the time series as the forecast for the next period.
  - The average changes, or moves, as new observations become available.
  - The moving average calculation is

Moving Average = Σ(most recent n data values)/n

Using Smoothing Methods in Forecasting

- Weighted Moving Averages
  - This method involves selecting weights for each of the data values and then computing a weighted mean as the forecast.
  - For example, a 3-period weighted moving average would be computed as follows.

$$F_{t+1} = w_1(Y_{t-2}) + w_2(Y_{t-1}) + w_3(Y_t)$$

where the sum of the weights (w values) is 1.

Using Smoothing Methods in Forecasting

- Exponential Smoothing
  - It is a special case of the weighted moving averages method in which we select only the weight for the most recent observation.
  - The weight placed on the most recent observation is the value of the smoothing constant, α.
  - The weights for the other data values are computed automatically and become smaller at an exponential rate as the observations become older.

Using Smoothing Methods in Forecasting

- Exponential Smoothing

$$F_{t+1} = \alpha Y_t + (1 - \alpha)F_t$$

where F_{t+1} = forecast value for period $t + 1$

Y_t = actual value for period $t + 1$

F_t = forecast value for period t

α = smoothing constant ($0 \leq \alpha \leq 1$)

Example: Executive Seminars, Inc.

Executive Seminars specializes in conducting management development seminars. In order to better plan future revenues and costs, management would like to develop a forecasting model for their "Time Management" seminar.

Enrollments for the past ten "TM" seminars are:

	(oldest)									(newest)
Seminar	1	2	3	4	5	6	7	8	9	10
Enroll.	34	40	35	39	41	36	33	38	43	40

Example: Executive Seminars, Inc.

- Exponential Smoothing

Let $\alpha = .2$, $F_1 = Y_1 = 34$

$F_2 = \alpha Y_1 + (1 - \alpha)F_1$

$= .2(34) + .8(34)$

$= 34$

$F_3 = \alpha Y_2 + (1 - \alpha)F_2$

$= .2(40) + .8(34)$

$= 35.20$

$F_4 = \alpha Y_3 + (1 - \alpha)F_3$

$= .2(35) + .8(35.20)$

$= 35.16$

. . . and so on

Example: Executive Seminars, Inc.

Seminar	Actual Enrollment	Exp. Sm. Forecast
1	34	34.00
2	40	34.00
3	35	35.20
4	39	35.16
5	41	35.93
6	36	36.94
7	33	36.76
8	38	36.00
9	43	36.40
10	40	37.72
11	Forecast for the next seminar =	38.18

Using Trend Projection in Forecasting

- Equation for Linear Trend

$$T_t = b_0 + b_1 t$$

where

T_t = trend value in period t

b_0 = intercept of the trend line

b_1 = slope of the trend line

t = time

Note: t is the independent variable.

Using Trend Projection in Forecasting

- Computing the Slope (b_1) and Intercept (b_0)

$$b_1 = \frac{\Sigma t Y_t - (\Sigma t \, \Sigma Y_t)/n}{\Sigma t^2 - (\Sigma t)^2/n}$$

$$b_0 = (\Sigma Y_t/n) - b_1 \Sigma t/n = \bar{Y} - b_1 \bar{t}$$

where

Y_t = actual value in period t

n = number of periods in time series

Example: Sailboat Sales, Inc.

Sailboat Sales is a major marine dealer in Chicago. The firm has experienced tremendous sales growth in the past several years. Management would like to develop a forecasting method that would enable them to better control inventories.

The annual sales, in number of boats, for one particular sailboat model for the past five years are:

Year	1	2	3	4	5
Sales	11	14	20	26	34

Example: Sailboat Sales, Inc.

- Linear Trend Equation

	t	Y_t	tY_t	t^2
	1	11	11	1
	2	14	28	4
	3	20	60	9
	4	26	104	16
	5	34	170	25
Total	15	105	373	55

Example: Sailboat Sales, Inc.

- Trend Projection

$$b_1 = \frac{373 - (15)(105)/5}{55 - (15)^2/5} = 5.8$$

$$b_0 = 105/5 - 5.8(15/5) = 3.6$$

$$T_t = 3.6 + 5.8t$$

$$T_6 = 3.6 + 5.8(6) = 38.4$$

Trend and Seasonal Components in Forecasting

- Multiplicative Model
- Calculating the Seasonal Indexes
- Deseasonalizing the Time Series
- Using the Deseasonalizing Time Series to Identify Trend
- Seasonal Adjustments
- Cyclical Component

Multiplicative Model

- Using T_t, S_t, and I_t to identify the trend, seasonal, and irregular components at time t, we describe the time series value Y_t by the following multiplicative time series model:

$$Y_t = T_t \times S_t \times I_t$$

- T_t is measured in units of the item being forecast.
- S_t and I_t are measured in relative terms, with values above 1.00 indicating effects above the trend and values below 1.00 indicating effects below the trend.

Calculating the Seasonal Indexes

1. Compute a series of n -period centered moving averages, where n is the number of seasons in the time series.
2. If n is an even number, compute a series of 2-period centered moving averages.
3. Divide each time series observation by the corresponding centered moving average to identify the seasonal-irregular effect in the time series.
4. For each of the n seasons, average all the computed seasonal-irregular values for that season to eliminate the irregular influence and obtain an estimate of the seasonal influence, called the seasonal index, for that season.

Deseasonalizing the Time Series

- The purpose of finding seasonal indexes is to remove the seasonal effects from the time series.
- This process is called deseasonalizing the time series.
- By dividing each time series observation by the corresponding seasonal index, the result is a deseasonalized time series.
- With deseasonalized data, relevant comparisons can be made between observations in successive periods.

Using the Deseasonalizing Time Series to Identify Trend

- To identify the linear trend, we use the linear regression procedure covered earlier; in this case, the data are the deseasonalized time series values.
- In other words, Y_t now refers to the deseasonalized time series value at time t and not to the actual value of the time series.
- The resulting line equation is used to make trend projections, as it was earlier.

Seasonal Adjustments

- The final step in developing the forecast is to use the seasonal index to adjust the trend projection.
- The forecast for period t, season s, is obtained by multiplying the trend projection for period t by the seasonal index for season s.

$$Y_{t,s} = I_s[b_0 + b_1(t)]$$

Example: Eastern Athletic Supplies

Management of EAS would like to develop a quarterly sales forecast for one of their tennis rackets.

Sales of tennis rackets is highly seasonal and hence an accurate quarterly forecast could aid substantially in ordering raw material used in manufacturing.

The quarterly sales data (000 units) for the previous three years is shown on the next slide.

Example: Eastern Athletic Supplies

Year	Quarter	Sales
1	1	3
	2	9
	3	6
	4	2
2	1	4
	2	11
	3	8
	4	3
3	1	5
	2	15
	3	11
	4	3

Example: Eastern Athletic Supplies

Year	Quarter	Sales	4-CMA	2-CMA
1	1	3		
	2	9		
	3	6	5.00	5.13
	4	2	5.25	5.50
2	1	4	5.75	6.00
	2	11	6.25	6.38
	3	8	6.50	6.63
	4	3	6.75	7.25
3	1	5	7.75	8.13
	2	15	8.50	8.50
	3	11	8.50	
	4	3		

Example: Eastern Athletic Supplies

Year	Quarter	Sales	2-CMA	Seas-Irreg
1	1	3		
	2	9		
	3	6	5.13	1.17
	4	2	5.50	0.36
2	1	4	6.00	0.67
	2	11	6.38	1.72
	3	8	6.63	1.21
	4	3	7.25	0.41
3	1	5	8.13	0.62
	2	15	8.50	1.76
	3	11		
	4	3		

Example: Eastern Athletic Supplies

Quarter	Seas-Irreg Values	Seas. Index
1	0.67, 0.62	0.65
2	1.72, 1.76	1.74
3	1.17, 1.21	1.19
4	0.36, 0.41	0.39
		Total = 3.97

Seas.Index	Adj. Factor	Adj.Seas.Index
0.65	4/3.97	.655
1.74	4/3.97	1.753
1.19	4/3.97	1.199
0.39	4/3.97	.393
		Total = 4.000

Example: Eastern Athletic Supplies

Year	Quarter	Sales	Seas.Index	Deseas.Sales
1	1	3	.655	4.58
	2	9	1.753	5.13
	3	6	1.199	5.00
	4	2	.393	5.09
2	1	4	.655	6.11
	2	11	1.753	6.27
	3	8	1.199	6.67
	4	3	.393	7.63
3	1	5	.655	7.63
	2	15	1.753	8.56
	3	11	1.199	9.17
	4	3	.393	7.63

Example: Eastern Athletic Supplies

- Trend Projection

$$T_t = 4.066 + .3933t$$

$$T_{13} = 4.066 + .3993(13) = 9.1789$$

Using the trend component only, we would forecast sales of 9,179 tennis rackets for period 13 (year 4, quarter 1).

Example: Eastern Athletic Supplies

- Seasonal Adjustments

Period *t*	Trend Forec.	Seasonal Index	Quarterly Forecast
13	9,179	.655	6,012
14	9,572	1.753	16,780
15	9,966	1.199	11,949
16	10,359	.393	4,071

Models Based on Monthly Data

- Many businesses use monthly rather than quarterly forecasts.
- The preceding procedures can be applied with minor modifications:
  - A 12-month moving average replaces the 4-quarter moving average.
  - 12 monthly, rather than 4 quarterly, seasonal indexes must be computed.
  - Otherwise, the procedures are identical.

Cyclical Component

- The multiplicative model can be expanded to include a cyclical component that is expressed as a percentage of trend.

$$Y_t = T_t \times C_t \times S_t \times I_t$$

- However, there are difficulties in including a cyclical component:
  - A cycle can span several (many) years and enough data must be obtained to estimate the cyclical component.
  - Cycles usually vary in length.

Regression Analysis

- One or more independent variables can be used to predict the value of a single dependent variable.
- The time series value that we want to forecast is the dependent variable.
- The independent variable(s) might include any combination of the following:
  - Previous values of the time series variable itself
  - Economic/demographic variables
  - Time variables

Regression Analysis

- An autoregressive model is a regression model in which the independent variables are previous values of the time series being forecast.
- A causal forecasting model uses other time series related to the one being forecast in an effort to explain the cause of a time series' behavior.

Regression Analysis

- For a function involving k independent variables, we use the following notation:

Y_t = value of the time series in period t

x_{1t} = value of independent variable 1 in period t

x_{2t} = value of independent variable 2 in period t

o

o

o

x_{kt} = value of independent variable k in period t

Regression Analysis

- In forecasting sales of refrigerators, we might select the following five independent variables:

x_{1t} = price of refrigerator in period t

x_{2t} = total industry sales in period t - 1

x_{3t} = number of new-house building permits in period t - 1

x_{4t} = population forecast for period t

x_{5t} = advertising budget for period t

Regression Analysis

- The ***n*** periods of data necessary to develop the estimated regression equation would appear as:

Period (t)	Time Series (Y_t)	Value of Independent Variables (x_{1t})	(x_{2t})	(x_{3t})	.	.	(x_{kt})
1	Y_1	x_{11}	x_{21}	x_{31}	.	.	x_{k1}
2	Y_2	x_{12}	x_{22}	x_{32}	.	.	x_{k2}
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
n	Y_n	x_{1n}	x_{2n}	x_{3n}	.	.	x_{kn}

Qualitative Approaches to Forecasting

- Delphi Method
  - It is an attempt to develop forecasts through "group consensus."
  - The goal is to produce a relatively narrow spread of opinions within which the majority of the panel of experts concur.
- Expert Judgment
  - Experts individually consider information that they believe will influence the variable; then they combine their conclusions into a forecast.
  - No two experts are likely to consider the same information in the same way.

Qualitative Approaches to Forecasting

- Scenario Writing
  - This procedure involves developing several conceptual scenarios, each based on a well-defined set of assumptions.
  - The decision maker must decide how likely each scenario is and then make decisions accordingly.
- Intuitive Approaches
  - A committee or panel seeks to develop new ideas or solve complex problems through a series of "brainstorming sessions."
  - Individuals are free to present any idea without being concerned about criticism or relevancy.

End of Chapter 18