Statistical Computing and Simulation

Spring 2017

Assignment 3, Due April 18/2017

1. For uniform (0,1) random variables  define  That is, *N* is the number of random numbers that must be summed to exceed 1.
   1. Estimate *E(N)* with standard errors by generating 1000, 2000, 5000, 10000, and 100000 values of *N*, and check if there are any patterns in the estimate and its s.e.
   2. Compute the density function of *N*, *E(N)*, and *Var(N)*.
2. Describe a rejection algorithm for generating normal and logistic distribution, i.e.,



1. (a) Test the generation methods of normal distribution introduced in class, i.e., Box-Muller, Polar, Ratio-of-uniform, and also the random number generators from R. Based on your simulation results, choose the “best” generator.

(b) In the class we mentioned it is found by several researchers that

*a* (multiplier) = 131

*c* (increment) = 0

*m* (modulus) = 2^35

would have X ∈ (–3.3,3.6), if plugging congruential generators into the Box-Muller method. Verify if you would have similar results.

1. Simulate the following simple model of auto insurance claims:

* Claims arise according to a Poisson process at a rate of 100 per year.
* Each claim is a random size following a gamma distribution with shape and rate parameters both equal to 2. This distribution has a mean of 1and a variance of 1/2. Claims must be paid by the insurance company as soon as they arise.
* The insurance company earns premiums at a rate of 105 per year, spread evenly over the year (i.e. at time *t* measured in years, the total premium received is 105*t*.)

Write R code to do the following:

* + 1. Simulate the times and amounts of all the claims that would occur in one year. Draw a graph of the total amount of money that the insurance company would have through the year, starting from zero: it should increase smoothly with the premiums, and drop at each claim time.
    2. Repeat the simulation 1,000 times, and estimate the following quantities:

(i) The expected minimum amount of money that the insurance company has.

(ii) The expected final amount of money that the insurance company would have.

1. (a) Let X and X1 be i.i.d. r.v.’s and let where  Prove that the correlation coefficient between X and Y is  Describe an algorithm for generating a pair of r.v.'s (X,Y) for which 

(b) Using the idea in (a), describe an algorithm for generating a random vector  where and

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1. (a) Write a function for performing Gauss Elimination on the linear equation (and are vectors) in R. Apply your function to find inverse of the matrix  and compare your results for using 6 and 8 decimal digits. (Hint: Use the command “*round(data,dig=6)*.”)

(b) You can use the function “*solve*” to check if your function is correct, where you can assign your own vector.