Statistical Computing and Simulation

Spring 2017

Assignment 2, Due March 28/2017

1. (a) Write a computer program using the Mid-Square Method using 6 digits to generate 10,000 random numbers ranging over [0, 999999]. Use the Kolmogorov-Smirnov Goodness-of-fit test to see if the random numbers that you create are uniformly distributed. (Note: You must notify the initial seed number used, and you may adapt 0.05 as the α value. Also, you may find warning messages for conducting the Goodness-of-fit test, and comment on the Goodness-of-fit test. )

(b) Consider the random number generator, i.e., the generator used by Vax before 1993. Use both the and Kolmogorov-Smirnov Goodness-of-fit tests to check if the data are from U(0,1) distribution. Compare the result with those in (a) & (b), and discuss your finding based on the comparison.

(c) In class, we often use simulation tools in R, such as “*sample*” or “*ceiling(runif)*,” to generate random numbers from 1 to *k*, where *k* is a natural number. Using graphical tools (such as histogram) and statistical tests to check which one is a better tool in producing uniform numbers between 1 and *k*. (Hint: You may check if the size of *k* matters by, for example, assigning *k* a small and big value.)

1. (a) Verify that the sequence of numbers from, there are no runs up of length greater than 4. Generate 1,000 random numbers and see if there are no runs with length greater than 4. Also, use graphical tools to explore the possible patterns and discuss if  is a good random number generator.

(b) Use to generate random numbers from the standard normal distribution (i.e., via inversion) and apply tools in Time Series Analysis (e.g., *acf* and *pacf*) to verify whether they satisfy the properties of random samples from the standard normal distribution.

1. There are several ways for checking the goodness-of-fit for empirical data. In specific, there are a lot of normality tests available in R. Generate a random sample of size 10, 50, and 100 from N(0,1) and t-distribution (with degrees 10 and 20) in R. You may treat testing random numbers from t-distribution as the power. For a level of significance α = 0.05 test, choose at least four normality tests in R (“nortest” module) to check if this sample is from N(0,1). Tests used can include the Kolmogorov-Smirnov test and the Cramer-von Mises test. Note that you need to compare the differences among the tests you choose.
2. (a) Write your own R programs to perform Gap test and Permutation test. Then use this program to test if the uniform random numbers generated from Minitab (or SAS, SPSS, Excel) and R are independent.

(b) Write a small computer program to perform Up-and-down test. Then use this program and uniform random numbers generated from R to check if the mean and variance for the number of runs derived by Levene and Wolfowitz (1944) are valid.

1.  can be used to approximate N(0,1) distribution, where  are random sample from U(0,1).
	1. Based on compare the results of the Chi-square test and the Kolmogorov-Smirnov test, and see if there are any differences.
	2. Design two tests of independence (which are not the same as you saw in class) and apply them on the random sample that you generate.
2. (Bonus!) There are no standard methods for testing independence of random numbers. Still, we can use computer simulation to check the methods introduced in class have larger powers for detecting dependence between observations. You may apply AR(1) or AR(2) to the observations, by allowing the correlation coefficients varying from 0 to 1. Note: For observations satisfy AR(1), we have, and.

