W4

ANOVA Analysis

Question 5

One-Way ANOVA: Productivity Enhancement

Experiment Overview & Data

Objective

Allied Corporation wants to increase the productivity of its line workers. Four different programs have been suggested to help increase productivity. Twenty employees, making up a sample, have been randomly assigned to one of the four programs and their output for a day's work has been recorded. You are given the results below.

Program Output (Units)

A	B	C	D
82	75	92	84
85	78	95	88
88	80	90	85
84	77	94	86
86	80	94	87
Avg: 85	Avg: 78	Avg: 93	Avg: 86

$$: SSTR = \sum n_j(\bar{x}_j - \bar{x}_G)^2$$ +

$$SSE = \sum_{j=1}^{k}\sum_{i=1}^{n_j}(x_{ij} - \bar{x}_j)^2$$

=

$$SST = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(X_{ij} - \bar{X}_{..})^2$$

ANOVA Results & Statistical Hypotheses

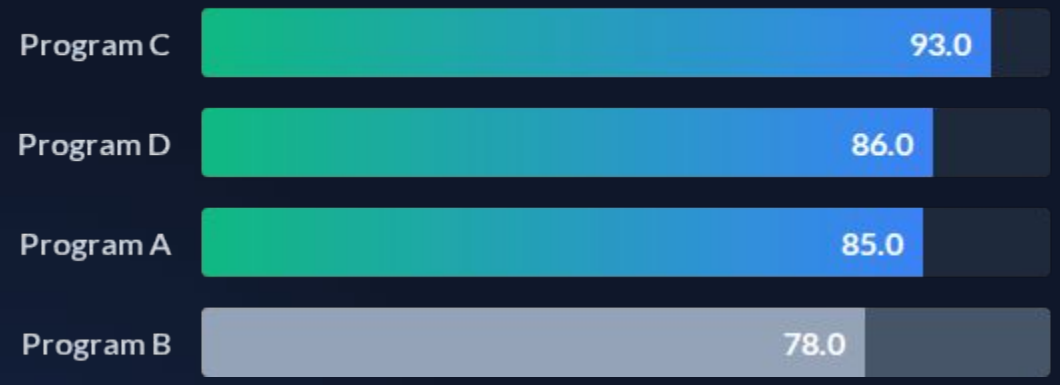
$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$ H_a : Not all means are equal

Source of Variation	Sum of Squares (SS)	df	Mean Square (MS)	F-Statistic
Treatments (Between)	565.0	3	188.33	47.08
Error (Within)	64	16	4	-
Total	629	19	-	-

Conclusion: Since F (32.47) > F-critical (3.24), we reject the null hypothesis at α = .05.

Mean Comparisons (Fisher's LSD)

Daily Mean Output



LSD Calculation:

$$LSD = t_{.025}\sqrt{MSE\left(\frac{2}{n}\right)} \approx 2.68$$

Any pair of means differing by more than 2.68 is statistically different.

Finding: Program C is significantly superior to all other programs.

Program B is significantly inferior to all others.

Question 6

Randomized Block Design: Penicillin Manufacture

A process for the manufacture of penicillin was being investigated, and yield was the response of primary interest. There were 4 variants of the basic process to be studied, denoted as treatment A, B, C, and D. It was known that an important raw material, corn steep liquor, was quite variable. Fortunately blends sufficient for four runs could be made, thus applying the opportunity to run all 4 treatments with each of 5 blocks.

RBD: Treatment vs. Raw Material Variability

Accounting for variability in "Corn Steep Liquor" by blocking raw material blends.

Block (Blend)	A	B	C	D	Block Avg
1	90	89	98	95	93.0
2	85	78	93	80	84.0
3	82	88	88	86	86.0
4	88	93	90	85	89.0
5	80	82	81	89	83.0
Tr. Avg	85.0	86.0	90.0	87.0	G=87.0

*Adjusted numbers for calculation practice.

RBD Analysis Results

3.64

F-Statistic for Treatments

變異來源	平方和 (SS)	自由度 (df)	平均平方 (MS)	F 檢定值
處理 (Treatments)	70	3	23.33	1.24
集區 (Blocks)	264	4	66	3.5
誤差 (Error)	226	12	18.83	-
總計 (Total)	560	19	-	-

F-Critical (3, 12) = 3.49.

Conclusion: 1.24<3.49. do not Reject null.

Comparison of Penicillin Variants

Pairwise Differences

Fisher's LSD for RBD (α=.05):

Variant C (90) vs Variant A (85): **5.0***

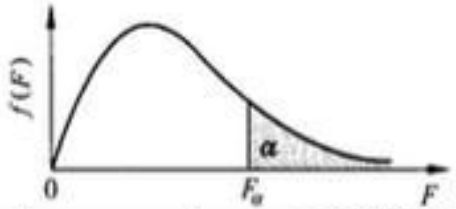
Variant C (90) vs Variant B (86): **4.0**

Variant C (90) vs Variant D (87): 3.0

*Significant difference detected.

$$LSD = t_{\alpha/2}\sqrt{MSE\left(\frac{2}{b}\right)} = 2.179\sqrt{18.83 \times \frac{2}{5}} \approx 2.179 \times 2.7447 \approx 5.98$$

附表五：F 分布临界值表（α =0.05）



V_2 \ V_1	1	2	3	4	5	6	8	10	15
1	161.4	199.5	215.7	224.6	230.2	234.0	238.9	241.9	245.9
2	18.51	19.00	19.16	19.25	19.30	19.33	19.37	19.40	19.43
3	10.13	9.55	9.28	9.12	9.01	8.94	8.85	8.79	8.70
4	7.71	6.94	6.59	6.39	6.26	6.16	6.04	5.96	5.86
5	6.61	5.79	5.41	5.19	5.05	4.95	4.82	4.74	4.62
6	5.99	5.14	4.76	4.53	4.39	4.28	4.15	4.06	3.94
7	5.59	4.74	4.35	4.12	3.97	3.87	3.73	3.64	3.51
8	5.32	4.46	4.07	3.84	3.69	3.58	3.44	3.35	3.22
9	5.12	4.26	3.86	3.63	3.48	3.37	3.23	3.14	3.01
10	4.96	4.10	3.71	3.48	3.33	3.22	3.07	2.98	2.85
11	4.84	3.98	3.59	3.36	3.20	3.09	2.95	2.85	2.72
12	4.75	3.89	3.49	3.26	3.11	3.00	2.85	2.75	2.62
13	4.67	3.81	3.41	3.18	3.03	2.92	2.77	2.67	2.53
14	4.60	3.74	3.34	3.11	2.96	2.85	2.70	2.60	2.46
15	4.54	3.68	3.29	3.06	2.90	2.79	2.64	2.54	2.40
16	4.49	3.63	3.24	3.01	2.85	2.74	2.59	2.49	2.35

自由度df	$t_{0.1}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$
1	3.078	6.314	12.706	31.821	63.656
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845

自由度df	$t_{0.1}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
31	1.309	1.696	2.040	2.453	2.744
32	1.309	1.694	2.037	2.449	2.738
33	1.308	1.692	2.035	2.445	2.733
34	1.307	1.691	2.032	2.441	2.728
35	1.306	1.690	2.030	2.438	2.724
36	1.306	1.688	2.028	2.434	2.719
37	1.305	1.687	2.026	2.431	2.715
38	1.304	1.686	2.024	2.429	2.712
39	1.304	1.685	2.023	2.426	2.708