

Time Series & Forecasting

Assignment Solutions & Statistical Analysis

Question 1: Sales Data

The following dataset reflects the sales volume of a specific product over the past 6 weeks. We will analyze this data using the exponential smoothing forecasting method.

Week	Sales Volume
1	120
2	110
3	115
4	125
5	118
6	122



1(a) Exponential Smoothing

Methodology

We use a smoothing constant of $\alpha = 0.2$. The forecast for the next period is calculated using the established formula:

$$F_{t+1} = \alpha Y_t + (1 - \alpha) F_t$$

To initialize the model, we set the forecast for the first week equal to its actual historical value:

$$F_1 = Y_1 = 120$$

Step-by-Step Results

Applying the formula iteratively to subsequent weeks:

$$F_2 = 0.2 (120) + 0.8 (120) = 120.00$$

$$F_3 = 0.2 (110) + 0.8 (120) = 118.00$$

$$F_4 = 0.2 (115) + 0.8 (118) = 117.40$$

$$F_5 = 0.2 (125) + 0.8 (117.4) = 118.92$$

$$F_6 = 0.2 (118) + 0.8 (118.92) = 118.74$$

週次	實際值 (Y)	預測值 (F)	誤差 (e)	誤差平方 (e ²)
2	110	120.00	-10.00	100.00
3	115	118.00	-3.00	9.00
4	125	117.40	+7.60	57.76
5	118	118.92	-0.92	0.85
6	122	118.74	+3.26	10.63

1(b) & 1(c) Error & Forecast

1(b) Mean Squared Error

The MSE is derived by squaring the forecast errors for weeks 2 to 6, summing them (SSE), and dividing by n=5.

$$SSE = (-10)^2 + (-3)^2 + \dots + 3.26^2$$

$$SSE = 178.26$$

$$MSE = \frac{178.26}{5} = 35.65$$

均方誤差公式

$$MSE = \frac{\sum (Y_t - F_t)^2}{n}$$

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1(c) Error & Forecast

1(c) Week 7 Forecast

Using the actual sales from Week 6 and our last calculated forecast, we can project the value for Week 7:

$$F_7 = 0.2 (Y_6) + 0.8 (F_6)$$

$$F_7 = 0.2 (122) + 0.8 (118.74)$$

$$F_7 = 119.39$$

1(d) Smoothing Constant

35.65
MSE ($\alpha = 0.2$)

Selecting the Best Alpha

When computing the Mean Squared Error with an alternative constant of $\alpha = 0.4$, the resulting MSE is **40.27**. Comparing the two findings:

- MSE for $\alpha = 0.2$ is 35.65
- MSE for $\alpha = 0.4$ is 40.27

Conclusion: The smoothing constant of $\alpha = 0.2$ is superior for this specific sales dataset because it generates a substantially lower Mean Squared Error, signifying a more precise historical fit.

Question 2: Seasonal Factors

2(a) Centered Moving Average

Year	Quarter	Sales	4-Q MA	CMA
1	1	100	-	-
1	2	120	-	-
1	3	150	120.00	121.25
1	4	110	122.50	123.75
2	1	110	125.00	127.50
2	2	130	130.00	131.25
2	3	170	132.50	135.00
2	4	120	137.50	140.00
3	1	130	142.50	145.00
3	2	150	147.50	150.00
3	3	190	152.50	-

2(b) Seasonal Ratios

Actual vs. CMA

By dividing the actual sales by the Centered Moving Average (CMA), we successfully isolate the seasonal variation component.

$$\text{Ratio} = \frac{\text{Actual}}{\text{CMA}}$$

For example, calculating the ratio for Year 1, Quarter 3 yields:

$$\frac{150}{121.25} = 1.237$$

2(b) Final Seasonal Factors

The raw quarterly averages are normalized by multiplying them by the adjustment factor ($4 / 3.996 = 1.001$).



Quarter 1

$$0.880 \times 1.001$$

0.881



Quarter 2

$$0.995 \times 1.001$$

0.996



Quarter 3

$$1.248 \times 1.001$$

1.249



Quarter 4

$$0.873 \times 1.001$$

0.874