統計學

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第二十一章：品質管制

http://csyue.nccu.edu.tw

Chapter Contents

Quality

The American Society for Quality (ASQ) defines quality as “*the characteristics of a product or service that bear on its ability to satisfy stated or implied needs*.”

Today, high-performing organizations:

- place emphasis on methods for monitoring and maintaining quality because they recognize quality as a fundamental need to be competitive in today’s global economy.
- changed the scope of their customer-driven focus to the development of broad-based corporate quality strategies, leading to the concept of **Total Quality (TQ)** (*see notes.)

> *Total Quality (TQ) is a people-focused management system that aims at continual increase in customer satisfaction at continually lower real cost. TQ is a total system approach (not a separate area or work program) and an integral part of high-level strategy.*
>
> *[TQ] works horizontally across function and departments, involves all employees, top to bottom, and extends backward and forward to include the supply chain and the customer chain. TQ stresses learning and adaptation to continual change as keys to organization.*

Introduction

Regardless of how it is implemented in different organizations, Total Quality is based on three fundamental principles:

- a focus on customers and stakeholders
- participation and teamwork throughout the organization
- a focus on continuous improvement and learning

In the first section of the chapter, we introduce the three quality management frameworks:

- the Malcolm Baldrige Quality Award, ISO 9001 standards, and the Six Sigma philosophy

In the last two sections, we introduce two statistical tools that can be used to monitor quality:

- statistical process control uses control charts to monitor a process and to determine whether corrective action should be taken to achieve the desired quality level.
- acceptance sampling is used in situations where a decision to accept or reject a group of items must be based on the quality found in a sample.

21.1 Quality Philosophies

Dr. Walter A. Shewhart developed a set of principles that are the basis for what is known today as process control.

Dr. Shewhart first developed a statistical control chart and changed the course of industrial history by bringing together the disciplines of statistics, engineering, and economics.

Dr. is recognized as the father of statistical quality control, and he was the first honorary Shewhart member of ASQ.

Dr. W. Edwards Deming (Dr. Shewhart's student) and Joseph Juran had a great influence on quality as they helped educate the Japanese on quality management shortly after World War II.

Dr. Deming stressed that the focus on quality must be led by managers and developed a list of 14 points he believed represent the key responsibilities of managers. Japan named its national quality award the *Deming Prize* in his honor (*see notes.)

Juran's approach to quality focused on three quality processes: quality planning, quality control, and quality improvement. He also proposed *fitness for use* as a simple definition of quality.

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Quality Philosophies (1 of 4)

Dr. Walter A. Shewhart

Developed a set of principles that are the basis for what is known today as process control (製程控制)

Constructed a diagram that would now be recognized as a statistical control chart

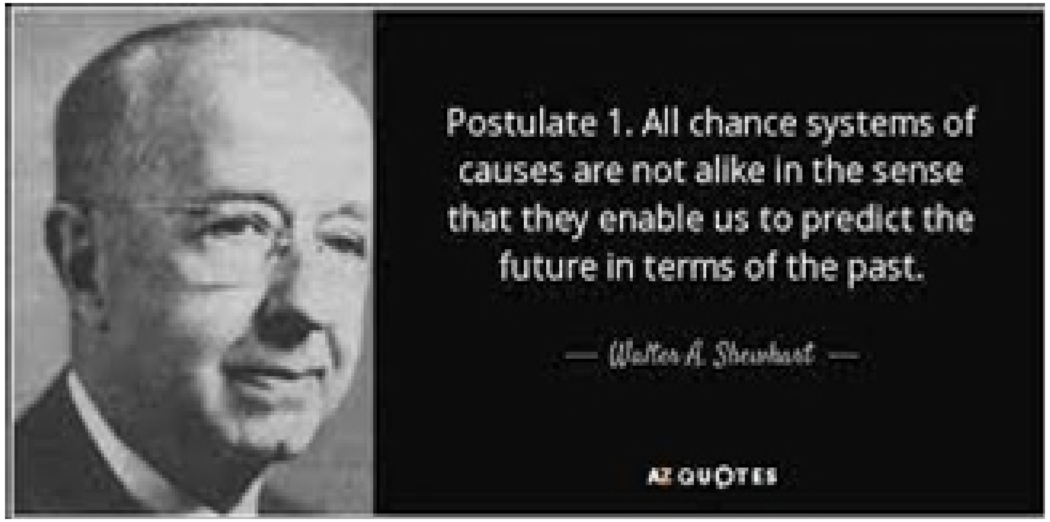
Brought together the disciplines of statistics, engineering, and economics and changed the course of industrial history

Recognized as the father of statistical quality control

First honorary member of ASQ

(American Society for Quality)

https://qualityleadershipblog.com/2020/08/02/dr-walter-shewhart-references/



Quality Philosophies (2 of 4)

Dr. W. Edwards Deming

Helped educate the Japanese on quality management shortly after World War II

Stressed that the focus on quality must be led by managers

Developed a list of 14 points he believed represent the key responsibilities of managers

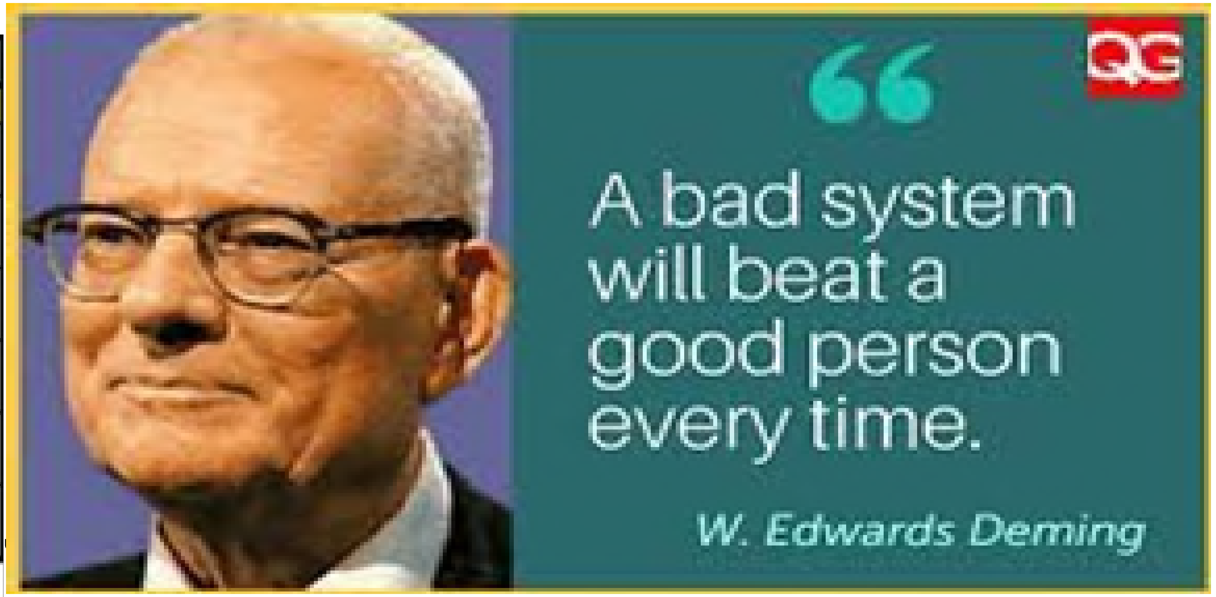
Japan named its national quality award the Deming Prize in his honor

https://www.qualitygurus.com/w-edwards-deming/

https://kaizeninstituteindia.files.wordpress.com/2013/10/14-points_demingresize1.jpg?w=487

Deming's 14 Points			
1	Create constancy of purpose .	8	Drive out fear.
2	Adopt the new philosophy and take on leadership .	9	Break down barriers. Work as a team.
3	Eliminate inspection. Build in quality.	10	Eliminate slogans. Fix the system.
4	Minimize total cost of by improving quality of supplies.	11	Eliminate quotas. Substitute Leadership
5	Constantly improve quality and productivity to decrease costs.	12	Remove barriers to pride of workmanship.
6	Institute training on the job.	13	Institute a vigorous program of education and self-improvement.
7	Supervision should be to help people.	14	The transformation is everybody's job.

Deming, *Out of the Crisis*, (p23-24)



Quality Philosophies (3 of 4)

Joseph Juran

- Helped educate the Japanese on quality management shortly after World War II
- Proposed a simple definition of quality: *fitness for use*
- His approach to quality focused on three quality processes: quality planning, quality control, and quality improvement

Quality Philosophies (4 of 4)

Other Significant Individuals

- Philip B. Crosby
- A.V. Feigenbaum
- Karou Ishikawa
- Genichi Taguchi (田口玄一)

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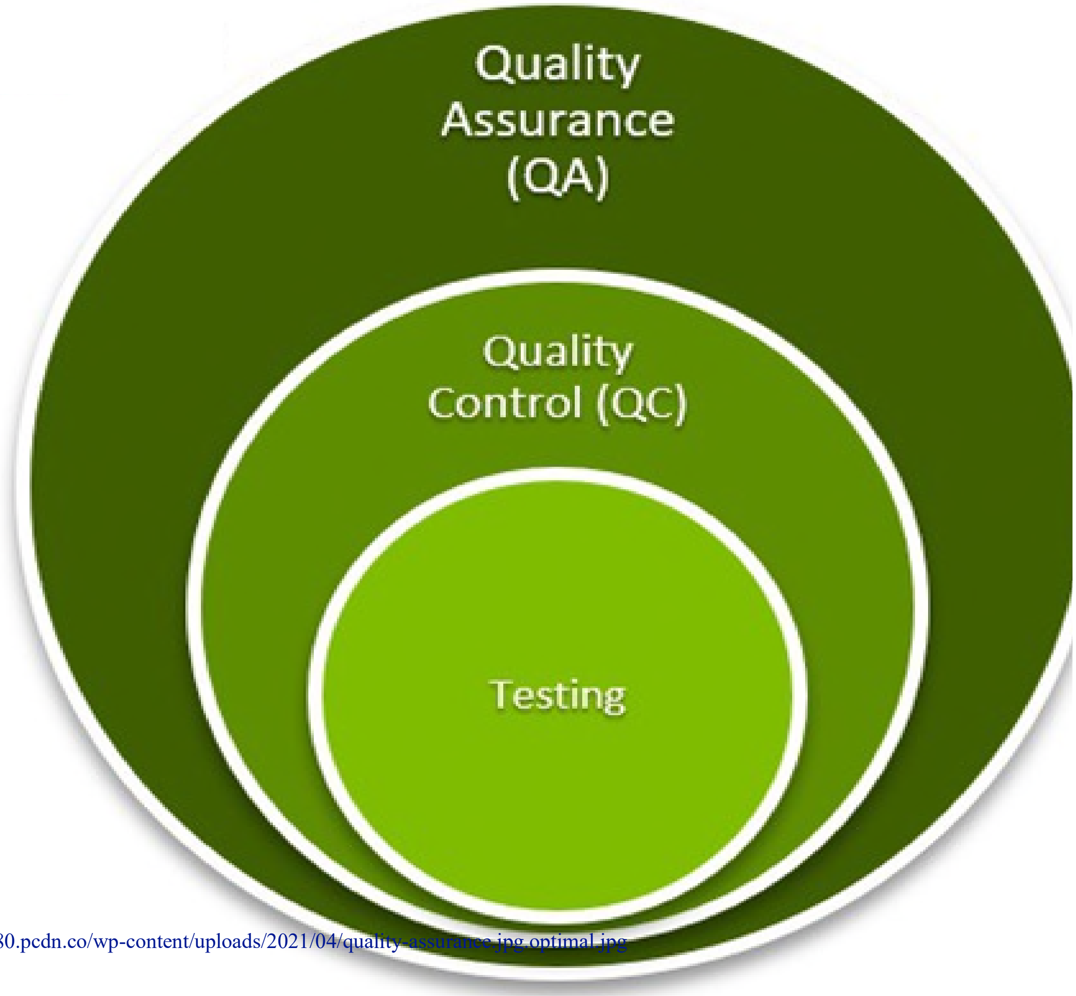
Quality Assurance vs. **Quality Control**

（品質保證）　　　（品質管制）



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21.1 Malcolm Baldrige National Quality Award

Established in 1987, the Malcolm Baldrige National Quality Award (*see notes) is given by the U.S. president to organizations that are judged to be outstanding in:

1. Leadership
2. Strategic Planning
3. Customer and Market Focus
4. Measurement, Analysis, and Knowledge Management
5. Human Resource Focus
6. Process Management
7. Business Results

The Award is managed by the U.S. Commerce Department’s National Institute of Standards and Technology (NIST.)

In 2003, the “Baldrige Index” (a hypothetical stock index comprised of Baldrige Award winning companies) outperformed the S&P 500 by 4.4 to 1.

21.1 ISO 9001

ISO 9001 defines the requirements and criteria that help organizations ensure that their products and services meet customers and regulatory requirements' quality standards.

ISO 9001 is the current evolution of ISO 9000, which was a series of five international standards published in 1987 by the International Organization for Standardization (ISO), Geneva, Switzerland.

Companies can use the standards defined in ISO 9001 to help determine what is needed to maintain an efficient quality conformance system.

ISO 9001 certification determines whether a company complies with its own quality system and defines the needs for

- an effective quality system
- ensuring that measuring and testing equipment is calibrated regularly
- maintaining an adequate record-keeping system

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21.1 Six Sigma

Six Sigma is a methodology, developed by Motorola in the late 1980s, aimed at improving the quality of products and services.

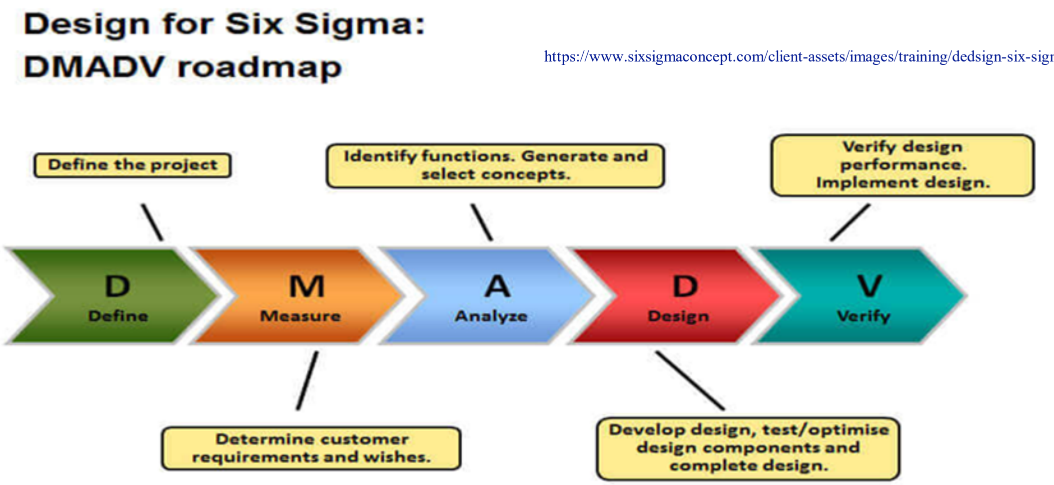
The Six Sigma level of quality has the goal to reduce the occurrence of defects to no more than 3.4 for every million opportunities.

An organization may undertake two kinds of Six Sigma projects:

- DMAIC (Define, Measure, Analyze, Improve, and Control) to help redesign existing processes
- DFSS (Design for Six Sigma) to design new products, processes, or services

In helping to redesign existing processes and design new processes, Six Sigma places a heavy emphasis on statistical analysis and careful measurement.

Today, Six Sigma is a major tool in helping organizations achieve Baldrige levels of business performance and process quality.



https://www.sixsigmaconcept.com/client-assets/images/training/dedsign-six-sigma.jpg

21.1 Six Sigma: Defects Per Million Opportunities

In Six Sigma terminology, a defect is any mistake or error that is passed on to the customer.

The Six Sigma process defines quality performance as defects per million opportunities (dpmo.)

Using Excel, we can show that, for a normally distributed process, 99.9999998% of the process output will be within ±6 standard deviations (σ) of the mean.

$$= \text{NORM.S.DIST}(6,\text{TRUE}) - \text{NORM.S.DIST}(-6,\text{TRUE}) = 0.999999998$$

Motorola's early work on Six Sigma convinced them that a process mean can shift on average by as much as 1.5σ.

If, for example, a process mean increases by 1.5σ, then what used to be the upper limit at +6σ, now it is only 4.5σ away from the process mean. Using Excel, the probability of a defect is:

$$= 1 - \text{NORM.S.DIST}(4.5,\text{TRUE}) = 0.0000034$$

The number of defects per one million opportunities is 3.4; that is, the quality level is 3.4 dpmo.

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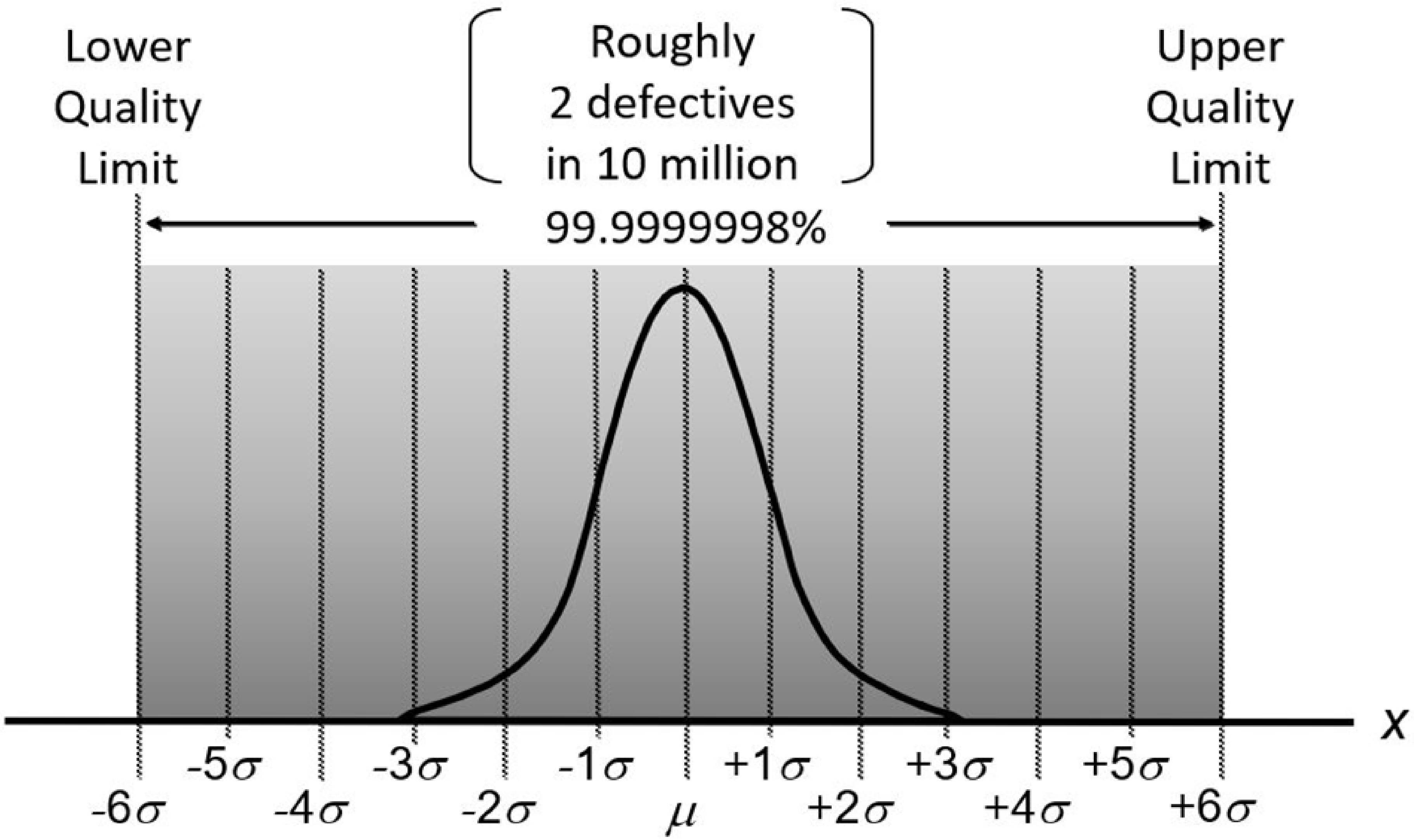
Quality Frameworks



21.1 Quality Terminology

Organizations that want to achieve and maintain a Six Sigma level of quality must emphasize methods for monitoring and maintaining quality.

Quality assurance refers to the entire system of policies, procedures, and guidelines established by an organization to achieve and maintain quality.

Quality assurance consists of two principal functions:

- *Quality engineering* has the objective to include quality in the design of products and processes and to identify quality problems prior to production.
- **Quality control** consists of a series of inspections and measurements used to determine whether quality standards are being met.
    - If quality standards are not being met, corrective or preventive action can be taken to achieve and maintain conformance.

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21.1 Quality in the Service Sector

While its roots are in manufacturing, quality control is also very important for businesses that focus primarily on providing services.

Examples of businesses that are primarily involved in providing services are health-care providers, law firms, hotels, airlines, restaurants, and banks.

Quality efforts in the service sector focus on ensuring customer satisfaction and improving the customer experience.

Services provided are often intangible, and because customer satisfaction is very subjective, it can be challenging to measure quality in services.

However, quality in service can be monitored by measuring the timeliness of providing service as well as by conducting customer satisfaction surveys and using customer loyalty cards.

Quality management in health care is also extremely important, and it focuses on programs that improve patient outcomes and reduce costly medical error in hospital and clinics.

21.2 Statistical Process Control

In **statistical process control**, a decision, based on the sampling and inspection of production output, is made to either continue the production process or adjust it to bring the items or goods being produced up to acceptable quality standards.

The main objective of statistical process control is to determine whether variations in output are due to assignable causes or common causes.

- **Assignable causes** are non-random variations in output due to tools wearing out, operator error, incorrect machine settings, poor quality raw material, and so on.
    - Whenever assignable causes are detected, we conclude that the process is *out of control*.
    - The producer should control assignable causes with corrective actions to bring the process back in control and to an acceptable level of quality.
- **Common causes** are randomly occurring variations in materials, humidity, temperature, and so on, that the producer cannot control and do not need to be adjusted.

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21.2 The Outcomes of Statistical Process Control

Statistical process control procedures are based on hypothesis-testing methodology.

- The null hypothesis, H_0, is formulated in terms of the production process being in control.
- The alternative hypothesis, H_a, is formulated in terms of the production process being out of control.

The table shows that correct decisions to continue an in-control process and adjust an out-of-control process are possible.

However, as with other hypothesis testing procedures, both a Type I error (adjusting an in-control process) and a Type II error (allowing an out-of-control process to continue) are also possible.

	State of Production Process	
Decision	**H_0 True** **Process in Control**	**H_0 False** **Process Out of Control**
Continue Process	Correct decision	Type II error (allowing an out-of-control process to continue)
Adjust Process	Type I error (adjusting an in-control process)	Correct decision

21.2 Control Charts

Statistical process control uses graphical displays known as control charts to monitor a production process.

Control charts provide a basis for deciding whether the variation in the output is due to common causes (in control) or assignable causes (out of control.)

Whenever an out-of-control situation is detected, adjustments or other corrective action will be taken to bring the process back into control.

All control charts include three lines:

a center line, an *upper control limit* (UCL), and a *lower control limit* (LCL)

UCL and LCL are chosen so that when the process is in control, there will be a high probability that the values of the measure of interest will be between the two lines.

Values outside of the control limits provide strong evidence that the process is out of control.

As the process is sampled, data points are plotted on the control chart from left to right. For each new data point, we carry a hypothesis test to determine whether the process is in control.

21.2 $\bar{x}$ Chart: Process Mean and Standard Deviation Known

An $\bar{x}$ **chart** is used when the quality of the output is measured in terms of a quantitative variable such as length, weight, temperature, and so on.

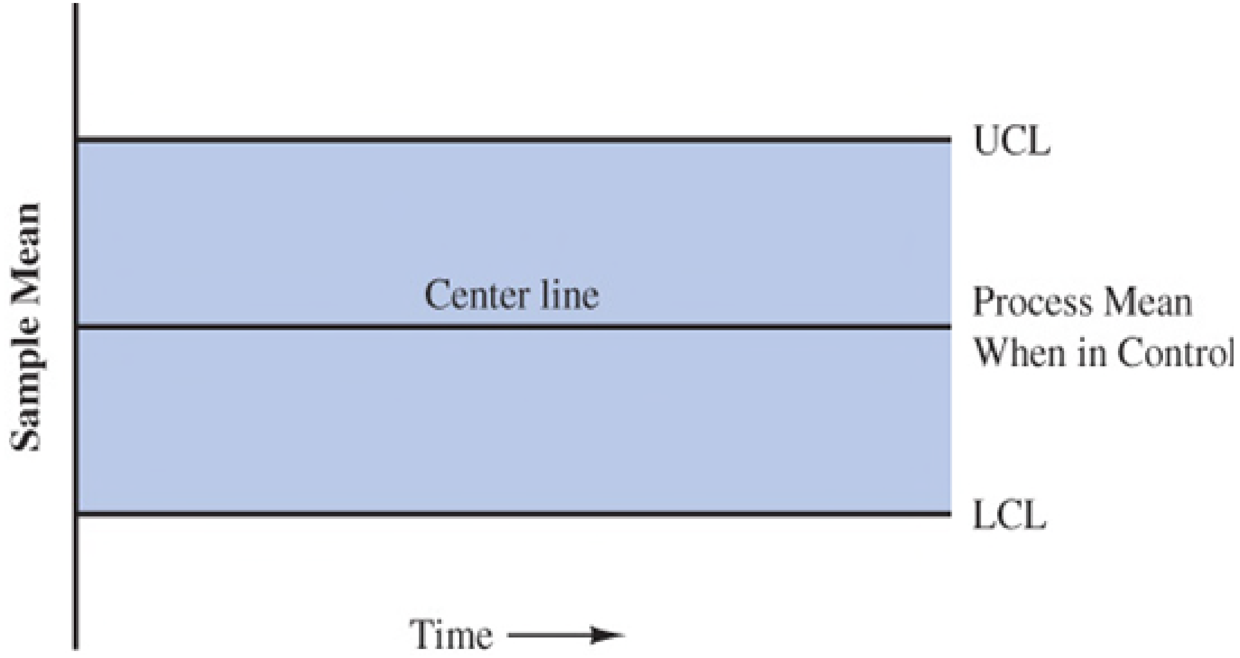
An $\bar{x}$ chart is used to monitor a production process based on the mean value found in a sample of the output.

The sampling distribution of $\bar{x}$ can be used to determine the variation that can be expected in the x values for a process that is in control.

If the variable of interest can be assumed normally distributed, the control limits are

$$\text{UCL} = \mu + 3\sigma_{\bar{x}} = \mu + 3\,\sigma/\sqrt{n}$$
$$\text{LCL} = \mu - 3\sigma_{\bar{x}} = \mu - 3\,\sigma/\sqrt{n}$$

Where, μ is the process mean, σ the process standard deviation, and n the sample size.



21.2 $\bar{x}$ Chart Application: μ and σ Known

The filling process of carton of cereals at KJW has mean $\mu = 16.05$ ounces and standard deviation $\sigma = 0.10$ ounces. We may assume that the filling weights are normally distributed.

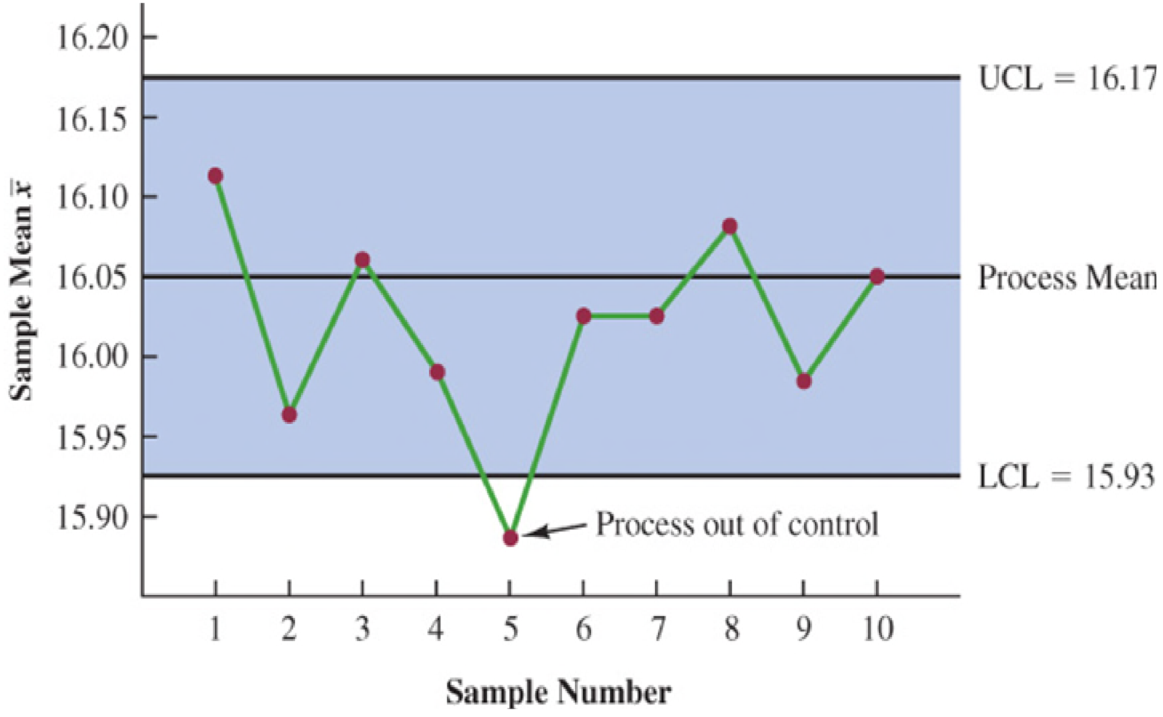
A quality control inspector periodically samples six cartons ($n = 6$) and uses the sample mean filling weight to determine whether the process is in control or out of control.

The standard error of the sampling distribution of $\bar{x}$ is calculated as $\sigma_{\bar{x}} = 0.10/\sqrt{6} = 0.04$.

Thus, the control limits are:

$$\text{UCL} = \mu + 3\sigma_{\bar{x}} = 16.05 + 3(0.04) = 16.17$$

$$\text{LCL} = \mu - 3\sigma_{\bar{x}} = 16.05 - 3(0.04) = 15.93$$



The $\bar{x}$ chart shows the fifth sample mean being below the LCL, indicating that assignable causes of output variation are present and that underfilling may be occurring (*see notes.)

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21.2 $\bar{x}$ Chart: Process Mean and Standard Deviation Unknown

When unknown, an estimate of the process mean, μ, is given by the overall sample mean, $\bar{\bar{x}}$.

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_k}{k}$$

Where, $\bar{x}_j$ is the mean of the jth sample, and k the number of samples.

When σ is unknown, it is common to monitor the variability of the process by using the range instead of the standard deviation because the range is easier to compute.

$$\bar{R} = \frac{R_1 + R_2 + \cdots + R_k}{k}$$ where, R_j is the range of the jth sample

An estimator of the process standard deviation σ can be then calculated as

$$\text{Estimator of } \sigma = \bar{R}/d_2$$

Where d_2 is a constant that depends on the sample size n, and it is described in the next slide.

21.2 Constants for $\bar{x}$ and R Control Charts

Observations in Sample, n	d_2	A_2	d_3	D_3	D_4	Observations in Sample, n	d_2	A_2	d_3	D_3	D_4
2	1.128	1.880	0.853	0	3.267	14	3.407	0.235	0.763	0.328	1.672
3	1.693	1.023	0.888	0	2.574	15	3.472	0.223	0.756	0.347	1.653
4	2.059	0.729	0.880	0	2.282	16	3.532	0.192	0.750	0.363	1.637
5	2.326	0.577	0.864	0	2.114	17	3.588	0.203	0.744	0.378	1.622
6	2.534	0.483	0.848	0	2.004	18	3.640	0.194	0.739	0.391	1.608
7	2.704	0.419	0.833	0.076	1.924	19	3.689	0.187	0.734	0.403	1.597
8	2.847	0.373	0.820	0.136	1.864	20	3.735	0.180	0.729	0.415	1.585
9	2.970	0.337	0.808	0.184	1.816	21	3.778	0.173	0.724	0.425	1.575
10	3.078	0.308	0.797	0.223	1.777	22	3.819	0.167	0.720	0.434	1.566
11	3.173	0.285	0.787	0.256	1.744	23	3.858	0.162	0.716	0.443	1.557
12	3.258	0.266	0.778	0.283	1.717	24	3.895	0.157	0.712	0.451	1.548
13	3.336	0.249	0.770	0.307	1.693	25	3.931	0.153	0.708	0.459	1.541

Source: Reprinted with permission from Table 27 of ASTM STP 15D, *ASTM Manual on Presentation of Data and Control Chart Analysis*, Copyright ASTM International, 100 Barr Harbor Drive, West Conshohocken, PA 19428.

21.2 The Jensen Computer Supplies Problem

Jensen Computer Supplies (JCS) manufactures solid-state drives and it just finished adjusting their production process so that it is operating in control.

A random sample of $n = 5$ drives each is selected every hour during production for a total of $k = 20$ samples, and the diameter x, of each sampled drive is recorded.

DATAfile: *Jensen*

Data computations shown to the right yield:

$$\bar{\bar{x}} = \sum \bar{x}_j / k = 69.9898/20 = 3.4995$$

$$\bar{R} = \sum \bar{R}_j / k = 0.5055/20 = 0.0253$$

Sample number	Observation 1	Observation 2	Observation 3	Observation 4	Observation 5	Sample Mean $\bar{x}_j$	Sample Range R_j
1	3.5056	3.5086	3.5144	3.5009	3.5030	3.5065	0.0135
2	3.4882	3.5085	3.4884	3.5250	3.5031	3.5026	0.0368
3	3.4897	3.4898	3.4995	3.5130	3.4969	3.4978	0.0233
4	3.5153	3.5120	3.4989	3.4900	3.4837	3.5000	0.0316
5	3.5059	3.5113	3.5011	3.4773	3.4801	3.4951	0.0340
6	3.4977	3.4961	3.5050	3.5014	3.5060	3.5012	0.0099
7	3.4910	3.4913	3.4976	3.4831	3.5044	3.4935	0.0213
8	3.4991	3.4853	3.4830	3.5083	3.5094	3.4970	0.0264
9	3.5099	3.5162	3.5228	3.4958	3.5004	3.5090	0.0270
10	3.4880	3.5015	3.5094	3.5102	3.5146	3.5047	0.0266
11	3.4881	3.4887	3.5141	3.5175	3.4863	3.4989	0.0312
12	3.5043	3.4867	3.4946	3.5018	3.4784	3.4932	0.0259
13	3.5043	3.4769	3.4944	3.5014	3.4904	3.4935	0.0274
14	3.5004	3.5030	3.5082	3.5045	3.5234	3.5079	0.0230
15	3.4846	3.4938	3.5065	3.5089	3.5011	3.4990	0.0243
16	3.5145	3.4832	3.5188	3.4935	3.4989	3.5018	0.0356
17	3.5004	3.5042	3.4954	3.5020	3.4889	3.4982	0.0153
18	3.4959	3.4823	3.4964	3.5082	3.4871	3.4940	0.0259
19	3.4878	3.4864	3.4960	3.5070	3.4984	3.4951	0.0206
20	3.4969	3.5144	3.5053	3.4985	3.4885	3.5007	0.0259

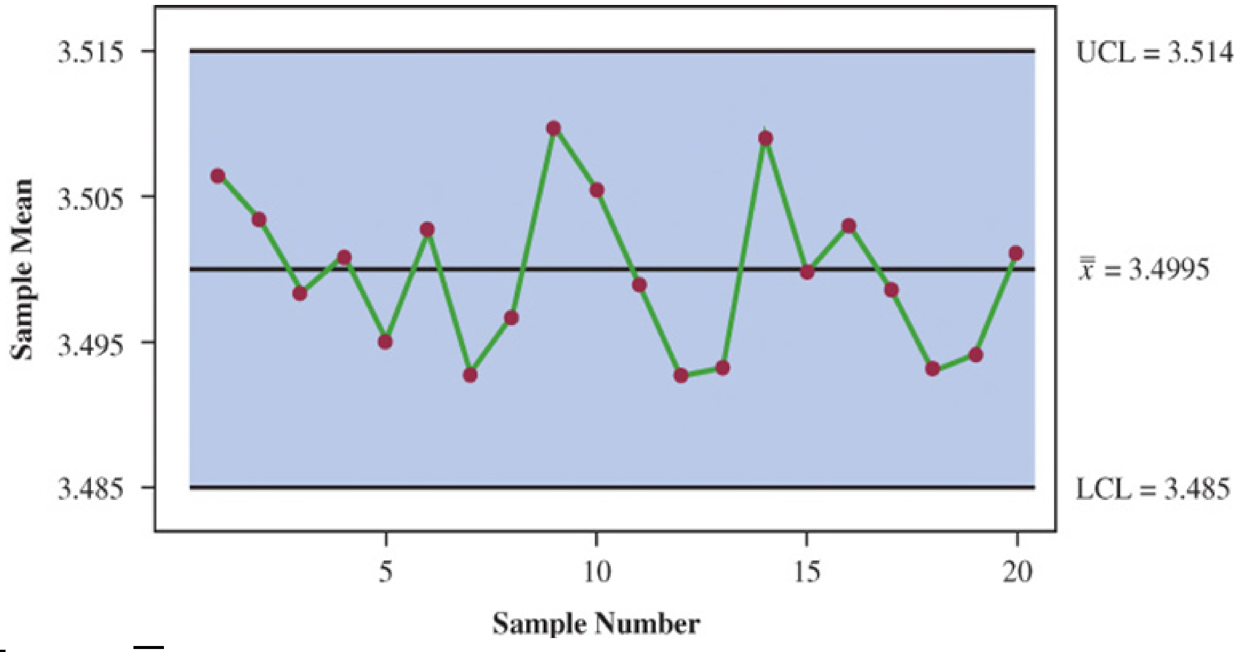
21.2 $\bar{x}$ Chart Application: μ and σ Unknown

We can now use the estimators of process mean, $\bar{\bar{x}} = 3.4995$, and range, $\bar{R} = 0.0253$, to estimate the standard deviation of the sampling distribution of $\bar{x}$ as

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\bar{R}/d_2}{\sqrt{n}}$$

Thus, the control limits are (*see notes):

$$\bar{\bar{x}} \pm 3\sigma_{\bar{x}} = \bar{\bar{x}} \pm 3\frac{\bar{R}/d_2}{\sqrt{n}} = \bar{\bar{x}} \pm \frac{3}{d_2\sqrt{n}}\bar{R} = \boldsymbol{\bar{\bar{x}} \pm A_2\bar{R}}$$



From the table of the constants for the $\bar{x}$ and R charts previously shown, for $n = 5$, we have

$$\bar{\bar{x}} \pm A_2\bar{R} = 3.4995 \pm 0.577(0.0253) = 3.4995 \pm 0.0146 = (3.485, 3.514)$$

The chart shows a process in statistical control.

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21.2 R Chart

A range chart (***R* chart**) can be used to control the variability of a process. If we think of the range of a sample as a random variable, we can describe it with mean and standard deviation.

The average range, $\bar{R}$, provides an estimate of the population mean range, and it can be shown that an estimate of the standard deviation of the range can be written as

$$\boldsymbol{\hat{\sigma}_R = d_3 \frac{\bar{R}}{d_2}}$$

Where d_2 and d_3 are constants that depend on the sample size as shown in the table of the constants for the $\bar{x}$ and R control charts. The control limits for the R chart are given by

$$\bar{R} \pm 3\hat{\sigma}_R = \bar{R} \pm d_3 \frac{\bar{R}}{d_2} = \bar{R}\left(1 \pm \frac{d_3}{d_2}\right) = \left[\bar{R}\left(1 - \frac{d_3}{d_2}\right), \bar{R}\left(1 + \frac{d_3}{d_2}\right)\right] = (\boldsymbol{\bar{R}D_3, \bar{R}D_4})$$

Where, $D_3 = 1 - \frac{d_3}{d_2}$ and $D_4 = 1 + \frac{d_3}{d_2}$, are also constants depending on the sample size.

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21.2 R Chart Application

In the Jensen Computer Supplies problem, the table of the constants for $\bar{x}$ and R control charts provides the constants $D_3 = 0$ and $D_4 = 2.114$, when $n = 5$.

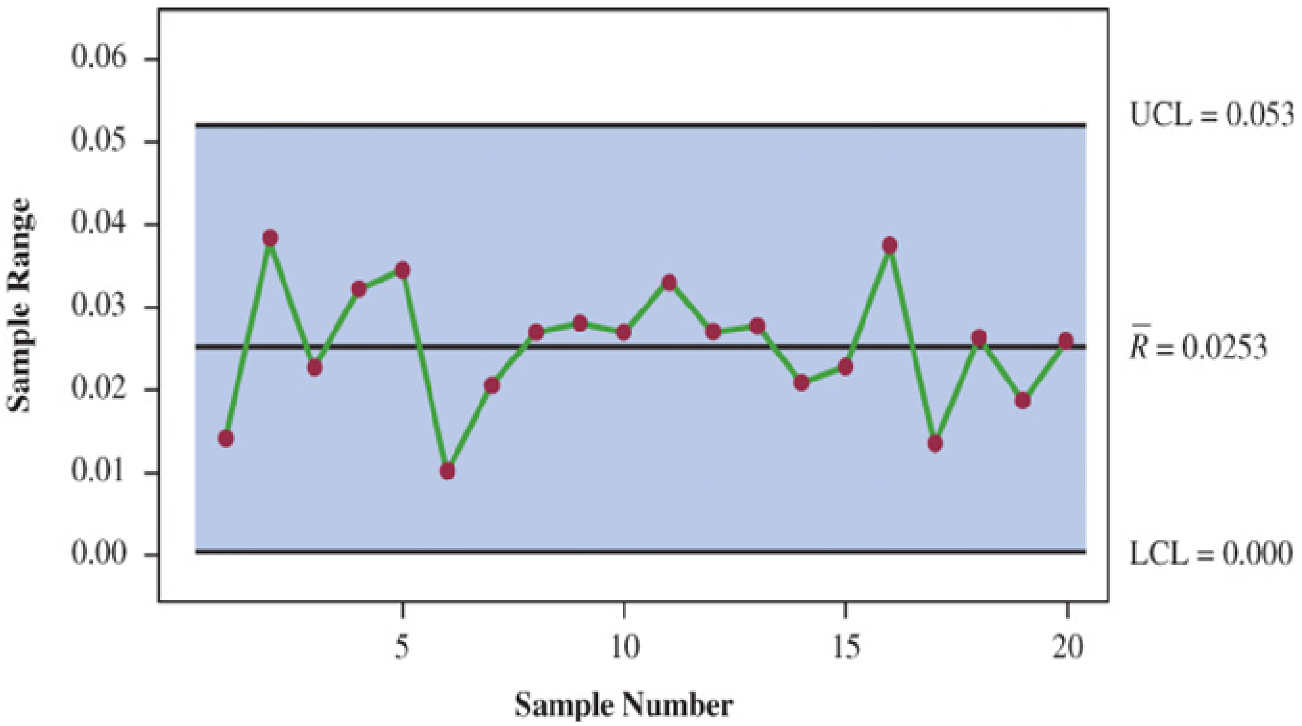
Thus, with $\bar{R} = 0.0253$, the control limits, UCL and LCL, are

$$\text{UCL} = \bar{R}D_4 = 0.0253(2.114) = 0.053$$

$$\text{LCL} = \bar{R}D_3 = 0.0253(0) = 0$$

The R Chart for the Jensen Computer Supplies problem has a center line $\bar{R} = 0.0253$, $\text{UCL} = 0.053$, and $\text{UCL} = 0$.

Because all plotted 20 sample ranges are within the control limits, we confirm that the process variability was in control during the sampling period.



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21.2 p Chart

A p **chart** is used when the quality of the output is measured in terms of nondefective or defective items.

A p chart is used to monitor a production process based on $\bar{p}$, the proportion of defective items found in a sample of the output.

The sampling distribution of $\bar{p}$ can be used to determine the variation that can be expected in $\bar{p}$ values for a process that is in control.

If the sample size is large enough so that $np \geq 5$ and $n(1-p) \geq 5$, the sampling distribution of $\bar{p}$ can be assumed normal, and the control limits are written as

$$\text{UCL} = \boldsymbol{p} + \mathbf{3}\boldsymbol{\sigma}_{\bar{\boldsymbol{p}}}$$

$$\text{LCL} = \boldsymbol{p} - \mathbf{3}\boldsymbol{\sigma}_{\bar{\boldsymbol{p}}} \quad \text{(or zero, if } \boldsymbol{LCL} < \mathbf{0}\text{)}$$

Where, for a sample size n, the standard error of the proportion is, $\boldsymbol{\sigma}_{\bar{\boldsymbol{p}}} = \sqrt{\dfrac{\boldsymbol{p}(\mathbf{1}-\boldsymbol{p})}{\boldsymbol{n}}}$

21.2 p Chart Application

Consider the use of automated mail-sorting machines at a post office that scan the zip codes on letters and divert each letter to its proper carrier route.

Assume that, when a mail-sorting machine is operating correctly, or in a state of control, 3% of the letters are incorrectly diverted.

Thus, p, the proportion of letters incorrectly diverted when the process is in control, is 0.03.

With $p = 0.03$ and $n = 200$, we have (*see notes)

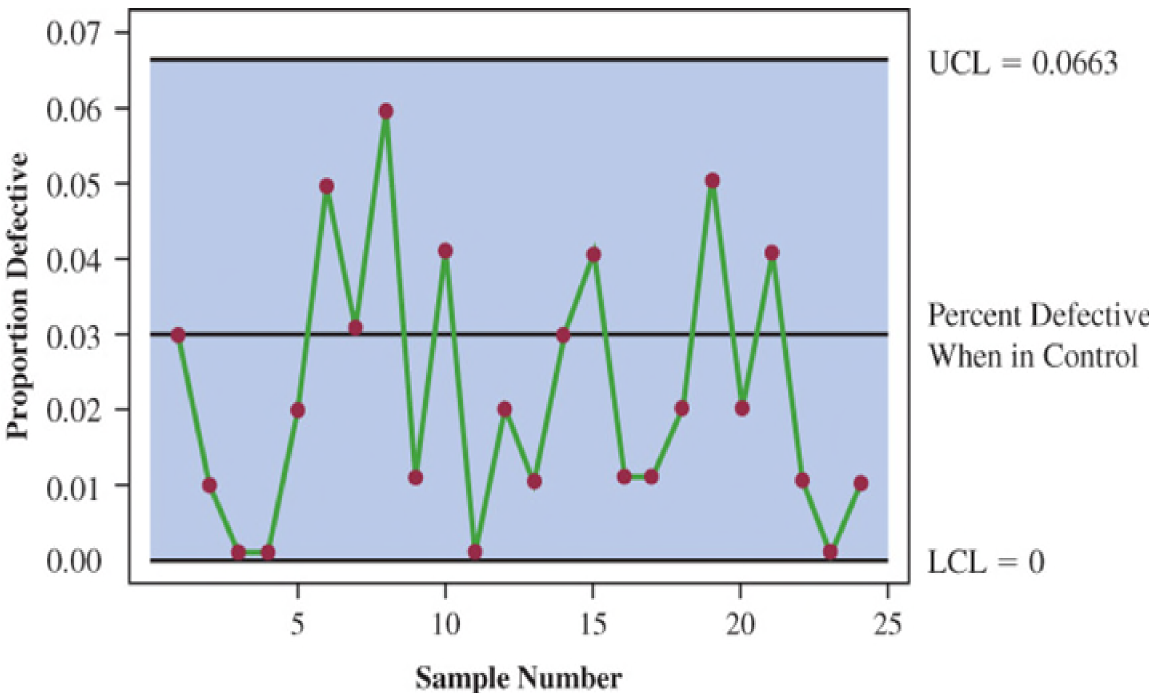
$$\sigma_{\bar{p}} = \sqrt{p(1-p)/n} = \sqrt{0.03(0.97)/200} = 0.0121$$

and the control limits are:

$$\text{UCL} = p + 3\sigma_{\bar{p}} = 0.03 + 3(0.0121) = 0.0663$$

$$\text{LCL} = p - 3\sigma_{\bar{p}} = max[0, 0.03 - 3(0.0121)] = 0$$

The chart shows a process in statistical control.



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21.2 np Chart

An $\boldsymbol{np}$ **chart** is a control chart for the number of defective items in a sample, where n is the sample size and p the probability of observing a defective item when the process is in control.

Whenever the sample size is large, that is, when $np \geq 5$ and $n(1-p) \geq 5$, the distribution of the number of defective items observed in a sample of size n can be approximated by a normal distribution with mean np and standard deviation $\sqrt{np(1-p)}$ (*see notes.)

In such case, the control limits for an np chart can be written as

$$\mathrm{UCL} = \boldsymbol{p} + \mathbf{3}\sqrt{\boldsymbol{np}(\mathbf{1}-\boldsymbol{p})}$$

$$\mathrm{LCL} = \boldsymbol{p} - \mathbf{3}\sqrt{\boldsymbol{np}(\mathbf{1}-\boldsymbol{p})} \quad \text{(or zero, if } \boldsymbol{LCL} < \mathbf{0}\text{)}$$

For the mail-sorting process example, with $p = 0.03$ and $n = 200$, the control limits are

$$\mathrm{UCL} = 6 + 3(2.4125) = 13.2375 \quad \text{and} \quad \mathrm{LCL} = 6 - 3(2.4125) = -1.2375$$

Because $\mathrm{LCL} < 0$, we set it to zero. Hence, if the number of letters diverted to incorrect routes is greater than 13 for any sample of 200, the process is concluded to be out of control.

21.2 Interpretation of Control Charts

The location and pattern of points in a control chart enable us to determine, with a small probability of error, whether a process is in statistical control.

A primary indication that a process may be out of control is a data point outside the control limits.

Finding such a point is statistical evidence that the process is out of control; in such cases, corrective action should be taken as soon as possible.

Certain patterns of points within the control limits can also be warning signals of quality problems:

- a large number of points on one side of center line
- six or seven points in a row that indicate either an increasing or decreasing trend.

When such a pattern occurs, the process should be reviewed for possible changes or shifts in quality. Corrective action to bring the process back into control may be necessary.

21.3 Acceptance Sampling

Acceptance sampling is a statistical method that enables us to base the accept-reject decision on the inspection of a sample of items from a group of items known as the lot.

Acceptance sampling has several advantages over the thorough testing of every component received through an approach known as 100% inspection:

- It is usually less expensive
- It causes less product damage due to less handling
- fewer inspectors are required
- It provides the only approach possible if destructive testing must be used

The statistical procedure of acceptance sampling uses the null and alternative hypotheses stated as follows

$$\text{H}_0\text{: Good}-\text{quality lot}$$

$$\text{H}_a\text{: Poor}-\text{quality lot}$$

21.3 Acceptance Sampling Procedure

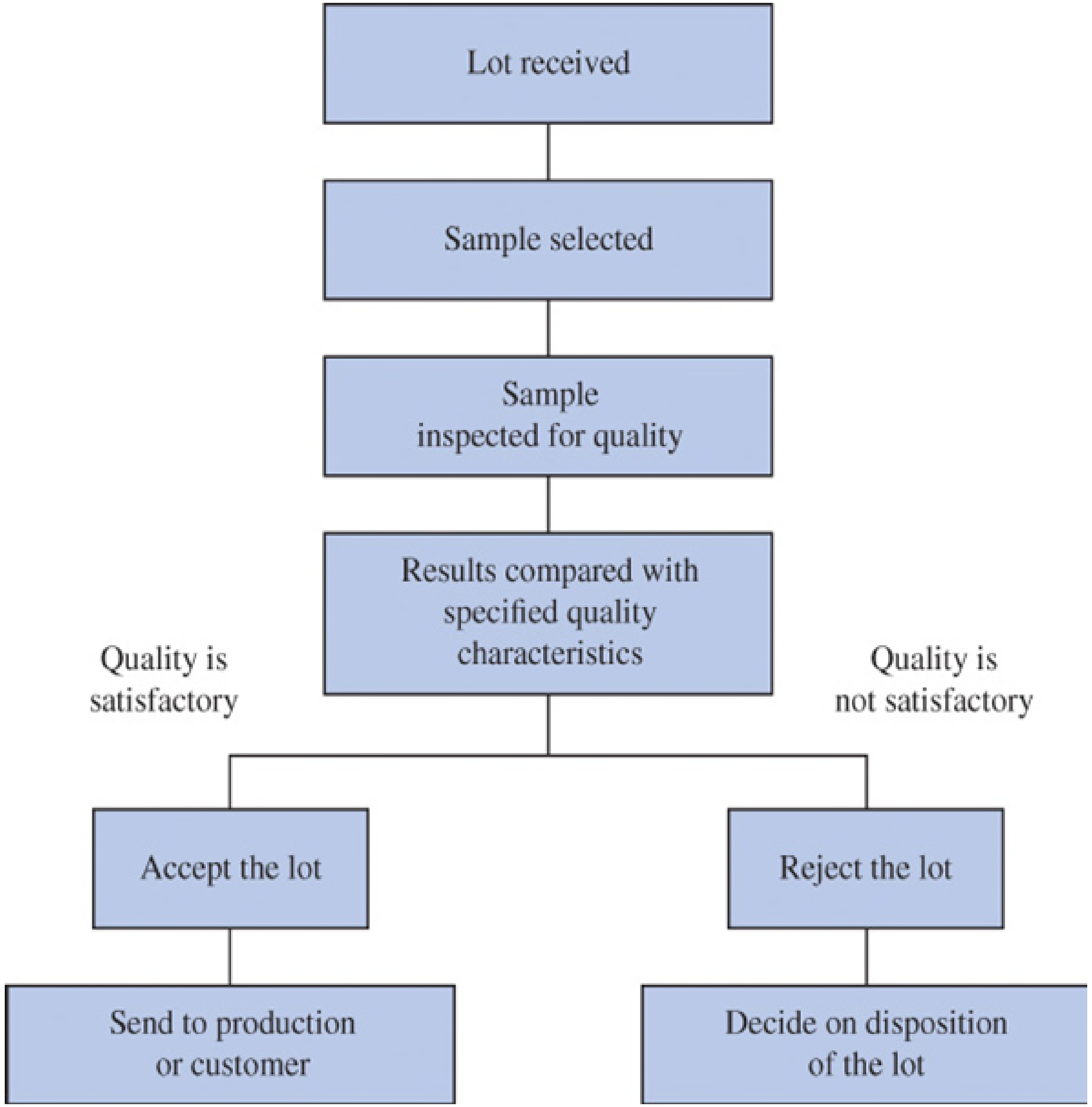
After a lot is received, a sample of items is selected for inspection and the results of the inspection are compared to specified quality characteristics.

- If the quality characteristics are satisfied, the lot is accepted into the supply chain.
- If the lot is rejected, managers must decide on its disposition.

At times, the decision may be to keep the lot and remove the unacceptable or nonconforming items.

In other cases, the entire lot may be returned to the supplier at the supplier's expense.

If the rejected lot consists of finished goods, the goods must be scrapped or reworked to meet acceptable quality standards.



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21.3 The Outcomes of Acceptance Sampling

With hypothesis testing, we must be aware of the possibilities of making a

- Type I error: rejecting a good-quality lot
- Type II error: accepting a poor-quality lot

Decision	State of the Lot: H_0 True Good-Quality Lot	State of the Lot: H_0 False Poor-Quality Lot
Accept the Lot	Correct decision	Type II error (accepting a poor-quality lot)
Reject the Lot	Type I error (rejecting a good-quality lot)	Correct decision

The probability of a Type I error is known as the **producer's risk**.

- For example, a producer's risk of 0.05 indicates a 5% chance that a good-quality lot will be erroneously rejected. Thus, creating a loss for the producer.

The probability of a Type II error is known as the **consumer's risk**.

- For example, a consumer's risk of 0.10 means a 10% chance that a poor-quality lot will be erroneously accepted. Thus, it may be used in production or shipped to the customer.

21.3 Acceptance Sampling Plan

An acceptance sampling plan consists of a sample size n and an acceptance criterion.

The **acceptance criterion c** is the maximum number of defective items that can be found in the sample and still indicate an acceptable lot.

The conclusions of the plan are based on the following decision rule of acceptance sampling:

- *Accept the lot* if $d \leq c$
- *Reject the lot* if $d > c$

Where d is the number of defective items found in the sample.

However, before implementing an acceptance sampling plan, the quality control inspector may want to evaluate the risks or errors possible under the plan.

The plan will be implemented only if both the producer's risk (Type I error) and the consumer's risk (Type II error) are controlled at reasonable levels.

Producer's and consumer's risk may be evaluated through a "what-if" type of analysis.

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21.3 An Application of Acceptance Sampling

KALI, a manufacturer of home appliances, is concerned about the quality of an overload protector purchased from a supplier, which is used in the assembly of home conditioners.

Rather than testing every component received through a time-consuming and expensive 100% inspection approach, KALI decides to use an acceptance sampling plan to monitor quality.

The manager of quality control selects a sample of $n = 15$ items from each incoming shipment, or lot, and states that the lot will be accepted only if no defective items ($c = 0$) are found.

After performing the tests, the quality control manager will reach a conclusion based on the following decision rule.

- *Accept the lot* if zero defective items are found ($d = 0$)
- *Reject the lot* if one or more defective items are found ($d \geq 1$)

Let us begin our analysis by evaluating the probabilities of Type I and Type II errors for KALI's acceptance sampling plan.

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21.3 Binomial Probability Function for Acceptance Sampling

For a shipment or lot in which 5% of the overload protectors are assumed to be defective, what is the probability that the $n = 15$, $c = 0$ sampling plan will lead us to accept the lot?

Because each overload protector tested will be either defective or nondefective, and because the lot size is large, the number of defective items in a sample of 15 is described by a binomial distribution probability function.

$$f(x) = \frac{n!}{x!\,(n-x)!} p^x (1-p)^{n-x}$$

Where

$n =$ the sample size

$p =$ the proportion of defective items in the lot

$x =$ the number of defective items in the sample

$f(x) =$ the probability of x defective items in the sample

21.3 Computing the Probability of Accepting a Lot

For a lot with 5% defective items ($p = 0.05$), the probability that the lot is accepted with x defects out of $n = 15$ in the KALI acceptance plan is

$$f(x) = \frac{15!}{x!\,(15-x)!}(0.05)^x(1-0.05)^{15-x}$$

The probability that zero overload protectors are defective ($x = 0$) and the lot is accepted is

$$f(0) = \frac{15!}{0!\,(15)!}(0.05)^0(1-0.05)^{15} = (0.95)^{15} = 0.4633$$

That is, the $n = 15$, $c = 0$ sampling plan has a 0.4633 probability of accepting a lot of with 5% defective items.

Thus, there must be a $1 - 0.4633 = 0.5367$ probability of a Type I error; that is, rejecting a lot with 5% defective items.

Using Excel's BINOM.DIST, we can quickly calculate the probability of accepting the lot for varying values of p.

Percent Defective in the Lot	Probability of Accepting the Lot
1	0.8601
2	0.7386
3	0.6333
4	0.5421
5	0.4633
10	0.2059
15	0.0874
20	0.0352
25	0.0134

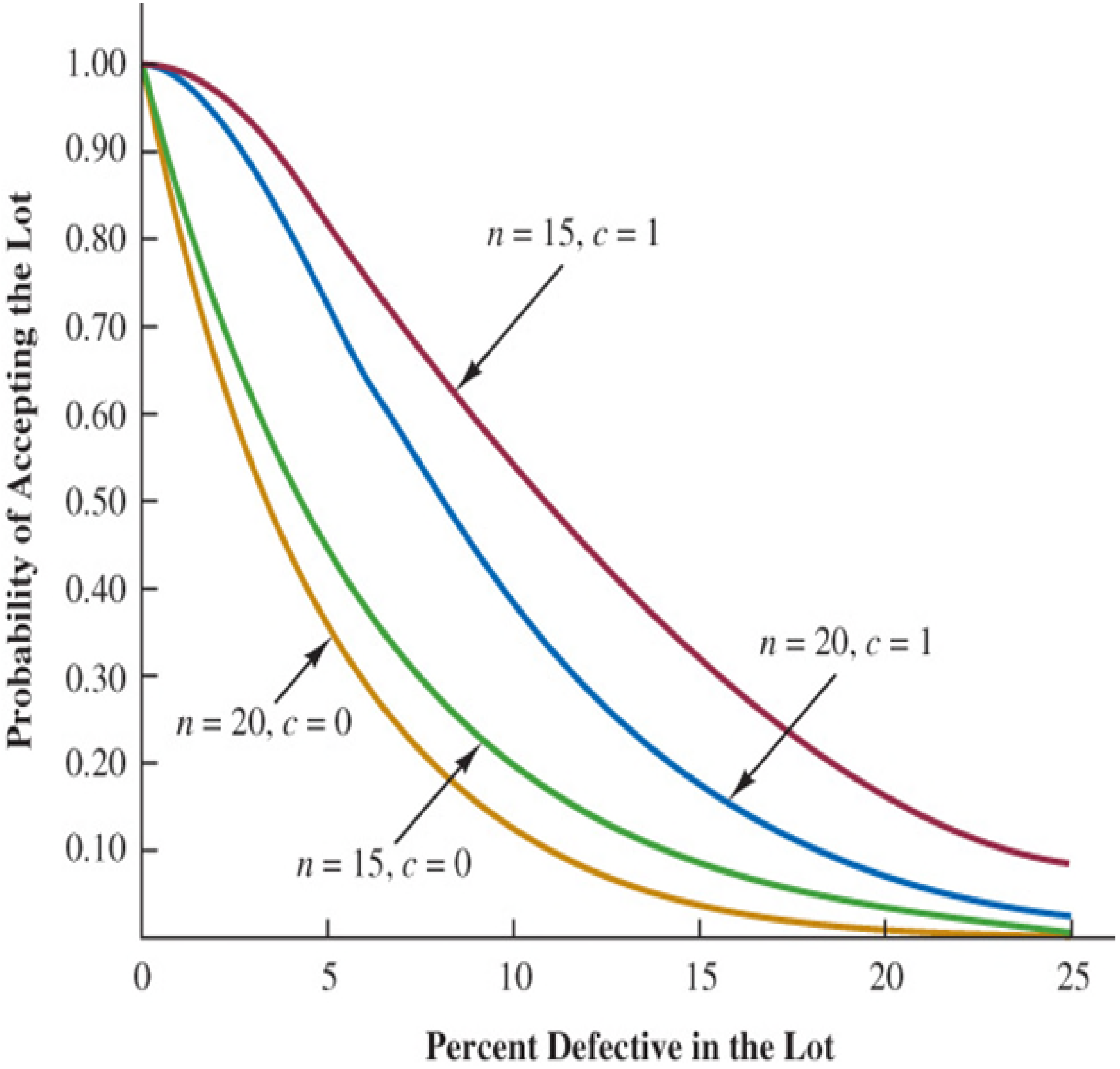
21.3 The Operating Characteristic (OC) Curve

The **operating characteristic (OC) curve** plots the probability of accepting the lot as a function of the percent of defective items.

The OC curves for four alternate KALI acceptance sampling plans ($n = 15$ or 20, and $c = 0$ or 1) are shown to the right.

The OC curve for the $n = 15$, $c = 0$, sampling plan probabilities, calculated in the previous slide, is depicted in green.

Note how the probability of accepting the lot for a given percent of defective items increases with both a *smaller* sample size, n, and a *larger* acceptance criterion, c.



21.3 Computing Type I and Type II Error Probabilities

Now that we know how to use the binomial distribution to compute the probability of accepting a lot with a given proportion defective, we are ready to select the values of n and c that determine the desired acceptance sampling plan for the application being studied

When formulating an acceptance sampling plan, the manager must specify two values for the proportion of defective items in the lot:

p_0, used to control for the producer's risk

p_1, used to control for the consumer's risk

For our calculations, we will use the following notation:

Producer's risk:

α = the probability that a lot with p_0 defectives will be rejected

Consumer's risk:

β = the probability that a lot with p_1 defectives will be accepted

21.3 Error Probabilities for the KALI Acceptance Sampling Plan

Suppose that the KALI managers specify $p_0 = 0.03$ and $p_1 = 0.15$.

We see from the OC curve previously shown that, for a $n = 15$, $c = 0$, acceptance sampling plan, we have

With $p_0 = 0.03$: $\alpha = 1 - f(0) = 1 - 0.6333 = 0.3667 \approx 0.37$

With $p_1 = 0.15$: $\beta = f(0) = 0.0874 \approx 0.09$

Suppose that the manager requests that the

producer's risk: $\alpha = 0.10$

consumer's risk: $\beta = 0.20$

Then, with a $n = 15$, $c = 0$, acceptance sampling plan, a $\alpha = 0.37 > 0.10$ producer's risk is unacceptable.

21.3 Selecting an Acceptance Sampling Plan

The OC curve to the right for the $n = 15$, $c = 0$, acceptance sampling plan depicts a visualization of the computations of α and β.

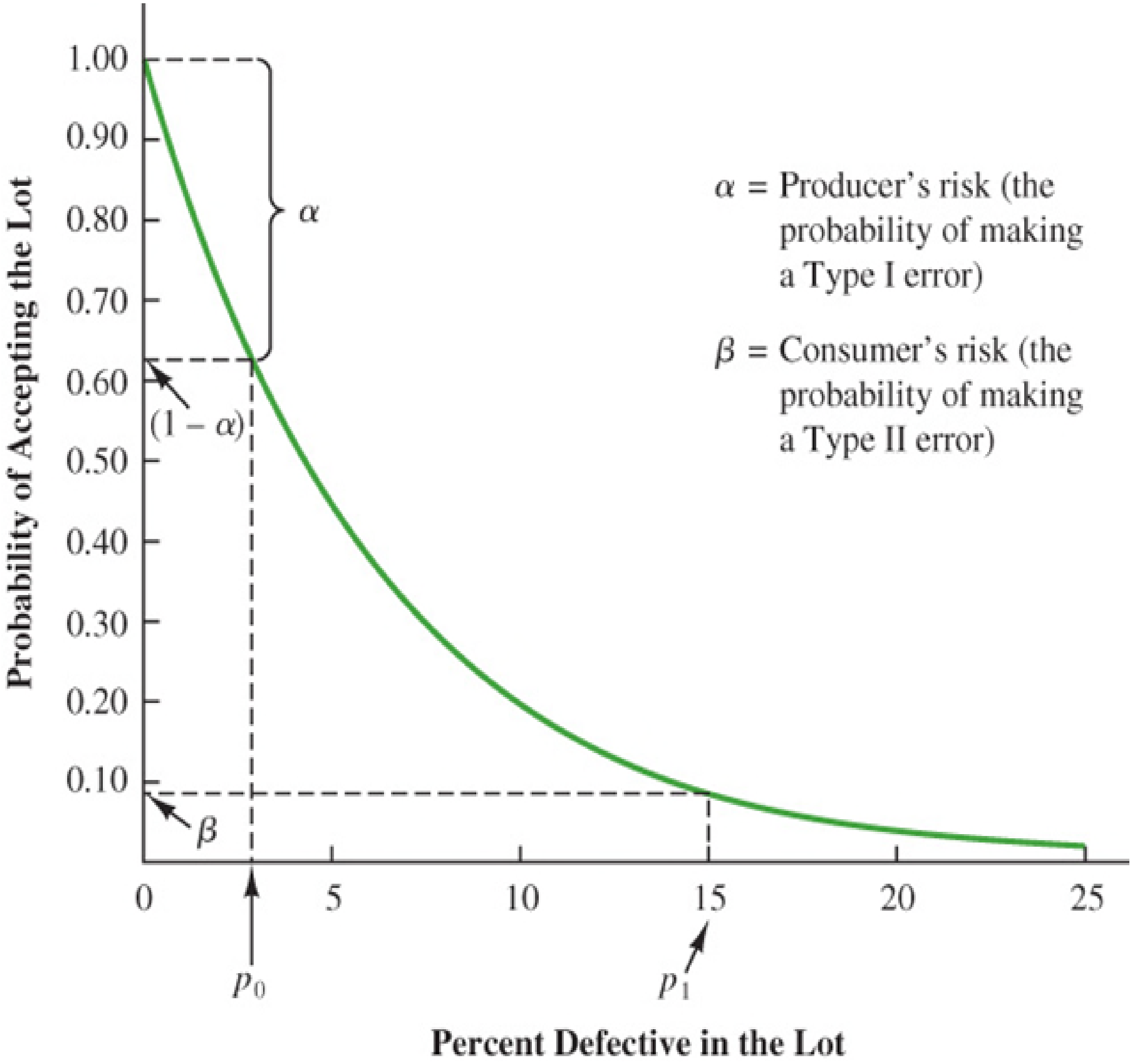
Because the producer's risk, $\alpha = 0.37$, is unacceptable to the quality control manager, a different sampling plan must be selected.

From the UC curves for different values of n and c shown earlier, we can see how the $n = 20$, $c = 1$, sampling plan comes closer to meeting both risk requirements.

$p_0 = 0.03$: $\quad \alpha = 1 - f(1) = 1 - 0.8802 \approx 0.12$

$p_1 = 0.15$: $\quad \beta = f(1) = 0.1756 \approx 0.18$

*See notes for additional remarks.



21.3 Multiple Sampling Plans

An alternative to the single-sampling plan such as the one used in the KALI example is a multiple sampling plan.

A **multiple sampling plan** uses two or more stages of sampling.

At each stage the decision possibilities are:

- stop sampling and accept the lot
- stop sampling and reject the lot
- continue sampling

Multiple sampling plans often result in a smaller total sample size than single-sample plans with the same Type I error and Type II error probabilities.

The next slide shows the logic of a two-stage, or double-sample, plan.

The development of the double-sample plan is more difficult because the sample sizes n_1 and n_1 and the acceptance criteria c_1, c_2, and c_3 must meet both the producer's and consumer's desired levels of acceptable risk.

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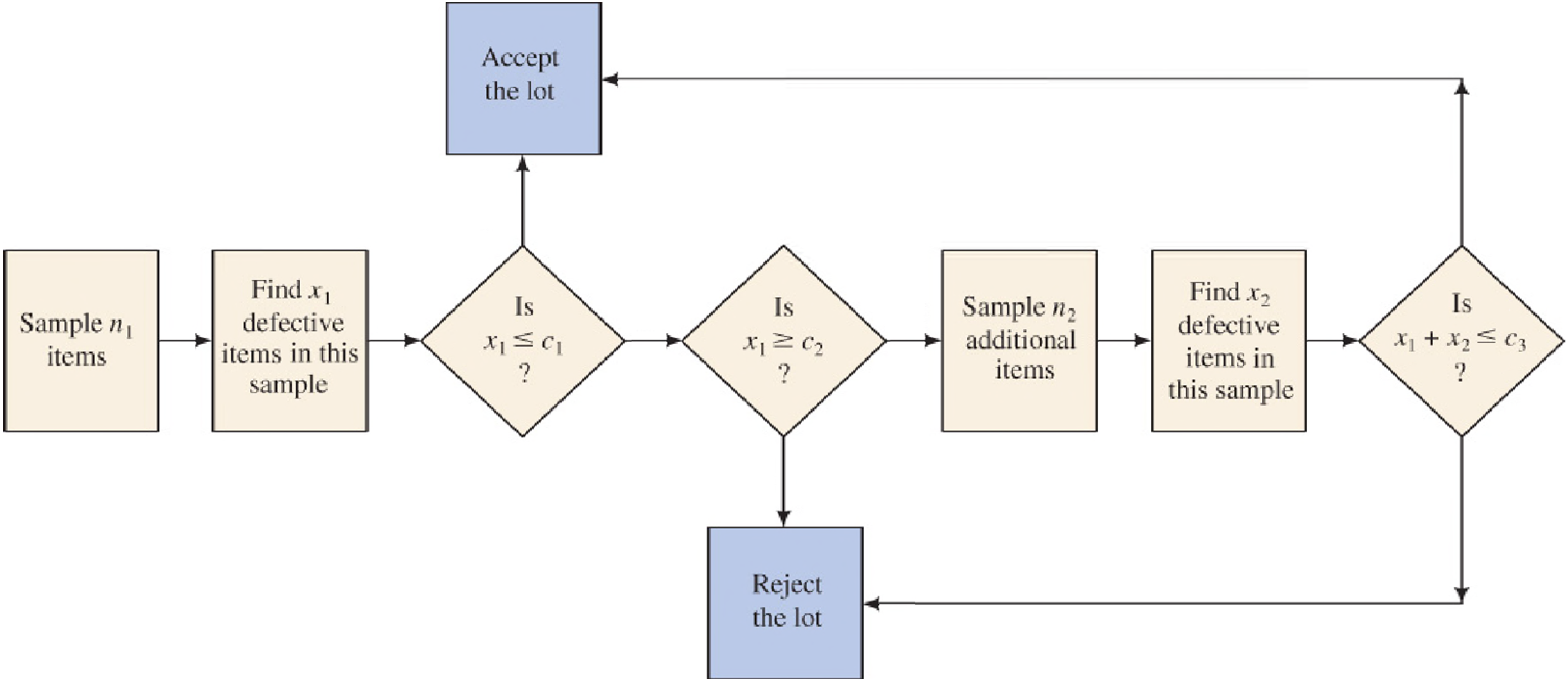
21.3 A Two-Stage Acceptance Sampling Plan



Summary

- In this chapter, we discussed how statistical methods can assist in the control of quality.
- We first introduced some quality-management philosophies such as Six Sigma.
- Then, we presented the $\bar{x}$, R, p, and np control charts to monitor process quality.
- Control limits are established for each chart; samples are selected periodically, and the data points plotted on the control chart.
  - Data points outside the control limits indicate that the process is out of control and that corrective action should be taken.
  - Patterns of data points within the control limits can also indicate potential quality control problems and suggest that corrective action may be warranted.
- Lastly, we discussed acceptance sampling, in which the number of defective items found in a sample selected from a lot provides the basis for accepting or rejecting the lot.
- The sample size and the acceptance criterion can be adjusted to control both the producer's risk (Type I error) and the consumer's risk (Type II error).

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