10/23統計學實習課

114-1統計學實習課

二項分布常態近似

114-1統計學實習課

二項分布常態近似、連續校正、常態查表

設X為一個二項分布隨機變數，代表在 20 次獨立試驗中成功的次數，每次成功的機率為 p=0.3, p=0.3。利用常態近似法（含連續校正）估計：P(4≤X≤8)

解答：

步驟 1：計算期望值與變異數

$$\mu = np = 20 \times 0.3 = 6$$

$$\sigma^2 = npq = 20 \times 0.3 \times 0.7 = 4.2$$

$$\sigma = \sqrt{4.2} \approx 2.05$$

步驟 2：使用連續校正

題目要求 $P(4 \leq X \leq 8)$，

所以在常態近似下轉換為： $P(3.5 \leq Y \leq 8.5)$

解答(續)：

步驟 3：標準化（轉為 Z 分數）

$$Z = \frac{Y - \mu}{\sigma} \quad \Rightarrow \quad Z_1 = \frac{3.5 - 6}{2.05} = \frac{-2.5}{2.05} \approx -1.22$$

$$Z_2 = \frac{8.5 - 6}{2.05} = \frac{2.5}{2.05} \approx 1.22$$

步驟 4：查標準常態分布表

$$P(Z \leq 1.22) = 0.8888$$

$$P(Z \leq -1.22) = 1 - 0.8888 = 0.1112$$

步驟 5：求差得出最終機率

$$P(4 \leq X \leq 8) \approx P(-1.22 \leq Z \leq 1.22) = 0.8888 - 0.1112 = 0.7776$$

第二次作業講解

114-1統計學實習課

作業講解—第一題(有改數字)

1. Corps of Cadets Students Joining the Military

Southeastern Ohio University offers its students the option of joining the Corps of Cadets to receive specialized leadership and military training while in college. Many students who join the Corps of Cadets in college choose to serve in the military after graduation, but not all Corps of Cadets students do so.

Suppose that **9%** of Southeastern Ohio University students elect to serve in the military after graduation. Of those students who elect to serve in the military after graduation, **82%** of them were part of the Corps of Cadets while in college. Of those who elect **not** to serve in the military after graduation, **6%** were part of the Corps of Cadets while in college.

(a) Given that a student is in the Corps of Cadets, what is the probability that the student will elect to join the military after graduation?
(b) Given that a student is **not** in the Corps of Cadets, what is the probability that the student will elect to join the military after graduation?
(c) If Southeastern Ohio University has **18,000** total students, approximately how many of these students are members of the Corps of Cadets?

作業講解—第二題(有改數字)

2. Automobile Insurance Damage Claims

The probability distribution for damage claims paid by the Newton Automobile Insurance Company on collision insurance follows:

Payment ($)	Probability
0	0.80
600	0.05
1,200	0.05
3,500	0.04
6,000	0.03
9,000	0.02
12,000	0.01

(a) Use the expected collision payment to determine the collision insurance premium that would enable the company to break even.

(b) The insurance company charges an annual rate of **$640** for the collision coverage. What is the expected value of the collision policy for a policyholder?

作業講解—第三題(有改數字)

3. Dog Food Marketing Focus Group

According to the American Veterinary Medical Association (AVMA), **41%** of households in the United States own a dog as a pet (AVMA website). Suppose that a company that sells dog food would like to establish a focus group to gather input on a new marketing campaign. The company plans to contact **30** randomly selected households to invite people to join the focus group.

(a) Compute the probability that **12** of these 30 households own a dog as a pet.
(b) Compute the probability that **3 or fewer** of these 30 households own a dog as a pet.
(c) For the sample of 30 households, compute the expected number of households who own a dog as a pet.
(d) For the sample of 30 households, compute the variance and standard deviation of households who own a dog as a pet.

作業講解—第四題(有改數字)

4. Computer Code Errors

The book *Code Complete* by Steve McDonnell estimates that there are 15 to 50 errors per 1000 lines of delivered code for computer programs. Assume that for a particular software package, the error rate is **30** per 1000 lines of code and that the number of errors per 1000 lines of code follows a **Poisson distribution**.

(a) What is the probability that a portion of code for this computer package that contains **300** lines of code contains no errors?

(b) What is the probability of there being exactly **25** errors in 1000 lines of code?

(c) What is the probability of more than **5** errors in 200 lines of code?

(d) What is the expected number of errors in a portion of code that contains **20,000** lines of code?

作業講解—第五題(有改數字)

5. Time to Complete Final Exam

The time needed to complete a final examination in a particular college course is normally distributed with a mean of **85 minutes** and a standard deviation of **12 minutes**.

(a) What is the probability of completing the exam in **70 minutes or less**?
(b) What is the probability that a student will complete the exam in more than **70 minutes but less than 80 minutes**?
(c) Assume that the class has **75 students** and that the examination period is **95 minutes** in length. How many students do you expect will be unable to complete the exam in the allotted time?

作業講解—第六題(有改數字)

6. Adults Who Smoke

The number of adults in the United States who smoke has declined greatly. In 2005 more than 20% of adults in the United States smoked, but this declined to about **13%** in 2022 (CDC.gov). Consider a group of **300** adults surveyed in the year 2022, and use the normal approximation of the binomial distribution to answer the questions below.

(a) What is the expected number of adults who smoke?
(b) What is the probability that fewer than **35** smoke?
(c) What is the probability that from **40 to 50** smoke?
(d) What is the probability that **60 or more** smoke?