



# 11/20 統計學實習課

114-1 統計學實習課

# 第三次作業講解—點估計與區 間估計

114-1統計學實習課

## 題目 3 :

A random sample of 12 final exam papers from a course was collected. The exam scores (out of 100) are:

**Scores:**

58, 73, 85, 90, 67, 48, 76, 82, 95, 61, 70, 88

A passing score in this course is **60**.

- (a) What is the point estimate of the mean exam score?
- (b) What is the point estimate of the proportion of students who passed the course?
- (c) Construct a **99% confidence interval** for the mean exam score.
- (d) Construct a **98% confidence interval** for the mean exam score.

## 解答 3 :

### (a) 平均分數的點估計

樣本共 12 筆，總分

$$58 + 73 + 85 + 90 + 67 + 48 + 76 + 82 + 95 + 61 + 70 + 88 = 893$$

平均：
$$\bar{x} = \frac{893}{12} \approx 74.42$$

### (b) 通過比例的點估計

60 分以上視為通過，共 10 人通過。

$$\hat{p} = \frac{10}{12} = 0.8333$$

### (c) 99% 信賴區間

先算樣本標準差（計算後得）： $s \approx 14.2921$

標準誤：
$$SE = \frac{s}{\sqrt{12}} \approx 4.1258$$

99% 信賴水準， $df = 11$ ： $t_{0.005,11} = 3.106$

邊際誤差： $ME = 2.718 \times 4.1258 \approx 11.214$

98% CI：

$$(74.42 - 11.21, 74.42 + 11.21) = (63.20, 85.63)$$

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### (d) 98% 信賴區間

98% 臨界值 :  $t_{0.01,11} = 2.718$

邊際誤差 :  $ME = 2.718 \times 4.1258 \approx 11.214$

98% CI :

$$(74.42 - 11.21, 74.42 + 11.21) = (63.20, 85.63)$$

## 題目 4 :

A simple random sample of **1,200** individuals shows that **38%** support candidate A. Construct a **95% confidence interval** for the population proportion.

## 解答 4 :

樣本比例 :

$$\hat{p} = 0.38$$

標準誤 :

$$SE = \sqrt{\frac{0.38(1 - 0.38)}{1200}} = \sqrt{\frac{0.2356}{1200}} \approx 0.0140$$

95% 臨界值為  $z = 1.96$ 。

邊際誤差 :

$$ME = 1.96 \times 0.0140 = 0.0274$$

95% CI :

$$0.38 \pm 0.0274 = (0.3526, 0.4074)$$

即約 35.26% ~ 40.74%。

## 題目 5 :

An aptitude test is normally distributed with population mean  $\mu = 78$  and population standard deviation  $\sigma = 10$ .

A simple random sample of **40** employees is selected from a population of **900**.

- (a) What are the expected value, standard deviation (including finite population correction), and the shape of the sampling distribution of  $\bar{X}$ ?
- (b) Find the probability that the sample mean falls between **75** and **82**.
- (c) Find the value  $C$  such that  $P(\bar{X} \geq C) = 0.03$ .

## 解答 5 :

(a) 抽樣分配期望、標準差、形狀

期望值 :

$$E(\bar{X}) = \mu = 78$$

無限母體標準誤 :

$$SE_{\infty} = \frac{10}{\sqrt{40}} = \frac{10}{6.3249} = 1.5811$$

有限母體修正 ( FPC ) :

$$FPC = \sqrt{\frac{900 - 40}{900 - 1}} = \sqrt{\frac{860}{899}} \approx 0.9744$$

真正標準差 :

$$\sigma_{\bar{X}} = 1.5811 \times 0.9744 \approx 1.5404$$

分配形狀 :

母體為常態 → 樣本平均仍為常態分配。

## 解答 5 :

(b) 計算  $P(75 \leq \bar{X} \leq 82)$

標準化 :

$$Z_1 = \frac{75 - 78}{1.5404} = -1.948, \quad Z_2 = \frac{82 - 78}{1.5404} = 2.597$$

查表得 :

$$\Phi(2.597) \approx 0.9953$$

$$\Phi(-1.948) \approx 0.0257$$

機率 :

$$0.9953 - 0.0257 = 0.9696$$

## 解答 5 :

(c) 求  $C$  使  $P(\bar{X} \geq C) = 0.03$

$$P(\bar{X} < C) = 0.97$$

查標準常態

$$z_{0.97} \approx 1.8808$$

代回 :

$$C = 78 + 1.8808(1.5404)$$

$$C \approx 78 + 2.898 = 80.90$$