



11/06統計學實習課

114-1統計學實習課



中央極限定理

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題目 1 :

At a large university, 25% of the students are majoring in Business. A random sample of 120 students is selected, and their majors are recorded.

Questions

- (a) Compute the standard error of the sample proportion.
- (b) What is the probability that the sample contains at least 35 Business majors?
- (c) What is the probability that the sample contains fewer than 28 Business majors?
- (d) What is the probability that the sample contains between 30 and 34 (inclusive) Business majors?

解答 1 :

At a large university, 25% of students are Business majors.

Sample size: $n = 120$, $p = 0.25$, $q = 1 - p = 0.75$

(a) Standard Error

$$SE = \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.25 \cdot 0.75}{120}} \approx 0.0395$$

Convert to Normal Approximation

$$\mu = np = 120(0.25) = 30, \quad \sigma = \sqrt{npq} = \sqrt{30 \cdot 0.75} \approx 4.74$$

(b) $P(X \geq 35)$

$$z = \frac{35 - 30}{4.74} \approx 1.055$$

$$P(X \geq 35) = 1 - \Phi(1.055) \approx 1 - 0.854 = 0.146$$

解答 1 :

(c) $P(X < 28)$

$$z = \frac{28 - 30}{4.74} \approx -0.422$$

$$P(X < 28) = \Phi(-0.422) \approx 0.336$$

(d) $P(30 \leq X \leq 34)$

$$z_{30} = \frac{30 - 30}{4.74} = 0 \Rightarrow \Phi(0) = 0.500$$

$$z_{34} = \frac{34 - 30}{4.74} \approx 0.844 \Rightarrow \Phi(0.844) \approx 0.800$$

$$P = 0.800 - 0.500 = 0.300$$

中央極限定理的應用

抽樣得到的「企業管理學生比例」 \hat{p}

原本 \hat{p} 的分配是 二項分配：

$$X \sim \text{Binomial}(n, p)$$

但因為 **n=120** 足夠大，而且：

$$np = 120(0.25) = 30 \geq 5, \quad n(1 - p) = 120(0.75) = 90 \geq 5$$

→ 可以用 常態分配去近似二項分配，也就是中央極限定理的應用。

所以才可以把它轉成：

$$X \approx N(np, np(1 - p))$$

並進一步換成 z 分數來算機率。

題目 2 :

A bank has analyzed the daily checking account balances of its customers and found that the population mean balance is \$520 and the population standard deviation is \$60. A random sample of 100 checking accounts is selected.

Questions

- (a) What is the probability that the sample mean is greater than \$530?
- (b) What is the probability that the sample mean is less than \$528?
- (c) What is the probability that the sample mean lies between \$522 and \$530?
- (d) What is the probability that the sample mean is at least \$515?

解答2：

$$\mu = 520, \quad \sigma = 60, \quad n = 100, \quad SE = \frac{\sigma}{\sqrt{n}} = \frac{60}{10} = 6$$

(a) $P(\bar{X} > 530)$

$$z = \frac{530 - 520}{6} = 1.667$$

$$P = 1 - \Phi(1.667) \approx 1 - 0.9525 = 0.0475$$

(b) $P(\bar{X} < 528)$

$$z = \frac{528 - 520}{6} = 1.333$$

$$P = \Phi(1.333) \approx 0.9082$$

解答2 :

$$(c) P(522 < \bar{X} < 530)$$

$$z_{522} = \frac{522 - 520}{6} = 0.333 \Rightarrow \Phi(0.333) \approx 0.630$$

$$z_{530} = \frac{530 - 520}{6} = 1.667 \Rightarrow \Phi(1.667) \approx 0.9525$$

$$P = 0.9525 - 0.630 = 0.3225$$

$$(d) P(\bar{X} \geq 515)$$

$$z = \frac{515 - 520}{6} = -0.833$$

$$P = 1 - \Phi(-0.833) = \Phi(0.833) \approx 0.7977$$

中央極限定理的應用

抽樣得到的帳戶平均金額 \bar{X}

母體本來可能不是常態（銀行帳戶餘額通常偏態），
但因為樣本量 $n = 100$ 已經很大，
中央極限定理保證：

$$\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

所以我們才能直接用常態分配和 z 分數計算：

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

換句話說，如果沒有中央極限定理，我們就不能直接用常態表來算機率。