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Selection effect modification to the Lee-Carter model

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Abstract

Although other risk factors can be used, depending on feasibility, marketing, and data availability, age and gender are the two most common risk factors considered in life insurance products. Previous studies have shown that the newly insured, who passed certain health examinations, tend to have lower mortality rates than those already insured. Insurance companies often use select and ultimate tables to handle mortality discrepancies between the insured in different policy years (i.e., the selection effect). However, the selection effect is easily confused with mortality improvement, and its estimate is likely to be influenced by the annual reduction in mortality rates. In this study, we propose modifying the Lee-Carter model, including the selection effect and mortality improvement. We first use a simulation to evaluate the parameter estimation of the proposed approach and then apply it to experienced data from Taiwan's largest insurance company, Cathay Life Insurance Company Ltd. The results of our simulation and empirical studies support the newly proposed approach, which provides stable and accurate estimates of the selection effect and mortality improvement. We also find that the size of the selection effect concerning policy year was larger than the difference in mortality rates between smokers and non-smokers; this is particularly noticeable for older age groups.

Keywords Lee–Carter model · Selection effect · Mortality risk · Mortality improvement · Simulation

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1 Introduction

Since the 1990s, a significant reduction in mortality has become a common concern worldwide. Mortality improvement would incur longevity risk, which refers to living longer, and pensions paid longer than expected. The longevity risk can result in financial insolvency for insurance companies. Therefore, mortality (in general), its trends, and factors affecting it have become the focus of various studies. Mortality models are often used to study mortality rates using a wide selection of variables. A few variables, such as weight, age, and sex,[1] were considered risk factors and used in the mortality models. However, it is challenging to collect data on these variables. Therefore, data availability, quality, and empirical analysis results restrict the use of these variables. Newly insured people are believed to have lower mortality rates than those already insured, and the mortality rates within a certain period, called the selection period, are often assumed to be an increasing or decreasing function of the policy year. This phenomenon is known as the selection effect.

The selection effect is a phenomenon of mortality that varies with time since joining a group is observed in various situations with different names, not restricted to life insurance. For example, the healthy worker effect indicates that workers usually have lower overall death rates, owing to employment selection [6, 8]. After marriage, the selection effect implies that married adults generally live longer than unmarried adults do [19, 20]. The adverse effects of retirement on health and longevity result from adverse selection.

Insurance companies use selected life tables to differentiate the mortality risk of the insured based on the policy year. The selection effect is well-known and often appears as a standard topic in actuarial mathematics textbooks [1, 5]. However, surprisingly, few studies have focused on modeling the selection effect. For example, Carriere [4] proposed a parametric model based on a linear combination of survival functions, whereas Renshaw and Haberman [13] used a mixture of generalized linear and nonlinear models. Richards [16] proposed a Hermite-spline approach to model the selection effect of mortality rates at post-retirement age. These studies focused mainly on the methodology for constructing the selected mortality tables, such as the connection between the mortality rates in different policy years. Additionally, the graduation methods used to construct select mortality tables are not common in practice [11]. A possible reason for the limited application of graduation methods may be that the graduation of mortality rates in two (i.e., age and policy year) or higher dimensions is more complicated than traditional one-dimensional graduation concerning age.

The unavailability of relevant data is another reason only a few methods for modeling the selection effect are found. A longer observation period is often required to evaluate whether the newly insured have lower mortality rates (recommended at least ten years or longer). It is challenging to accumulate sufficient exposure to insurance products with similar mortality profiles. This is probably a key reason for

[1] In 2011, the European Court of Justice ruled that using gender to calculate premiums and benefits was inconsistent with the European Charter.

Taiwan's lack of life tables. Another reason is the existence of a thorough and rigorous underwriting process. Notably, few select mortality tables are available globally, except in the U.S. and the U.K.; for example, since 1990–95 Basic Select and Ultimate Mortality Tables for Individual Life Insurance, the Society of Actuaries in the U.S. published three studies on select periods: 2001, 2008, and 2014 Valuation Basic Tables (VBT) [7]. In the U.K., select mortality tables are produced by a Continuous Mortality Investigation (CMI) [12]. The length of the selection period was different between insurers and insurance products, as reported in the 2014 Select Period Mortality Survey, which ranged from ten years to more than 30 years for term products. The size of the selection effect also varies significantly and is often too large to be ignored.

A longer observation period could further complicate the estimation of the selection effect. The mortality rates in later policy years can be affected by mortality reduction, and the selection effect may likely be underestimated[2] if the mortality rates of all ages decrease with time. To avoid underestimation: the selection effect and mortality improvement need to be considered. Most previous studies on selection effects have not incorporated mortality improvement.[3] It is still not considered for pricing products in many countries (including Taiwan) and not even for annuity products.

This study treats the selection effect as a risk factor and modifies the Lee-Carter (LC) model [9] to handle the mortality improvement and selection effect. We use a two-stage estimation: first, we estimate the LC model parameters, and then the selection effect. The estimation was conducted until it converged recursively. This procedure has been used in many modifications of the LC model. As expected, the estimates of both parameters were negatively biased at first and then became reasonably stable after a few iterations. We use a simulation to evaluate the proposed approach and verify whether it could provide unbiased parameter estimates. Furthermore, we use empirical data to demonstrate that the estimates of the selection effects are underestimated when mortality improvement is not considered.

The current proposed mortality model can identify mortality risk regarding policy years, and life insurance companies can use this value for underwriting and pricing products and calculating the reserves. This is especially important in Taiwan because all insurance companies must implement IFRS 17 before 2026.[4]

The remainder of this paper is organized as follows. Section 2 describes the proposed method and related literature. The simulation study presented in Sect. 3 evaluates the estimation properties of the proposed approach. The evaluations are based on standard assumptions, using a pre-specified selection table and applying Taiwan national population mortality data as a template. The empirical analysis described in Sect. 4 determines whether the proposed models have smaller estimation errors than

[2] Conversely, the selection effect would be overestimated if the insured were ex-smokers.

[3] Mortality improvement is an important factor for modeling mortality rates today; however, it was not considered until the late 1990s.

[4] IFRS 17 is an International Financial Reporting Standard, and is expected to be effective in 2023. In 2021, the Taiwan government decided to delay the effective date to 2026.

the LC model. Empirical data was obtained from the largest insurance company in Taiwan, Cathay Life Insurance Company Ltd. (CLI). The results from the simulation and empirical studies verify the benefit of using the proposed approach, which can provide stable and accurate estimates of the parameters and mortality rates. Finally, the conclusions are presented in Sect. 5.

2 Methodology

As mentioned in the previous section, the size and length of the selection effect vary considerably, depending on the merits of the insured population and insurance products. For example, the selection period is five years for smokers [17] in terms of life products, and the size of the selection effect is greater than 60% (i.e., the mortality rate is 40% or less compared to the standard group). Ideally, the results of estimating the size and length of the selection effect should not be influenced by the estimation methods, population size, the structure of the insured population, and other factors (e.g., lapse rate or underwriting). Intuitively, the grand averages of mortality and incidence[5] rates of different policy years can be used to estimate the size and length of the selection effect. However, mortality improvement distorts the estimation of the selection effects. Therefore, applying the LC [9] or other stochastic models is essential to reduce mortality. The LC model is a popular mortality model that has been used for more than 20 years to estimate and forecast mortality rates. The LC model assumes that:

$$\log(m_{xt}) = \alpha_x + \beta_x \kappa_t + \varepsilon_{xt}, \tag{1}$$

where m_{xt} is the central mortality rate for age x and time t. For error ε_{xt}, we apply the Poisson assumption [2], assuming that $D_{xt} \sim \text{Poisson}(E_{xt} \times m_{xt})$, where E_{xt} and D_{xt} are the exposure and number of deaths at age x and time t, respectively. We also assume Poisson errors for other modified LC models in this study. The LC model is similar to fitting a group of linear regression equations simultaneously with the same predictor (i.e., time), and each regression equation has its slope and intercept. The slope of the regression equation can be interpreted as an improvement in mortality over time for each age group. The time variable κ_t is usually assumed to be a random walk with a drift.

We propose adding the selection effect to the LC model, or

$$\log(m_{xts}) = \alpha_x + \beta_x \kappa_t + C_{xs} I\{s \le s_x\} + \varepsilon_{xts}, \tag{2}$$

where m_{xts} is the central mortality rate at age x, time t, and policy year s. Additionally, C_{xs} is the size of the selection effect at age x and policy year s, and s_x is the length of the select period at age x. In other words, the proposed model has three

[5] The selection effect can also exist in health insurance products. For example, cancer insurance is a popular product in Taiwan, and insurance claims occur when the insured is diagnosed with cancer for the first time. Thus, we can plug the incidence rates into the population's proposed model to evaluate a selection effect. Note that the proportion of initial disease cases is defined as an incidence rate.

coordinates (age, time, and policy year), similar to those of the cohort LC model [14]. However, the proposed approach does not face a problem of linear dependency between the three coordinates as in the cohort LC model where time = age + cohort. The estimation of our proposed approach is divided into two stages. First, we estimate the LC model's parameters, followed by an estimation of the selection effect. This two-stage estimation is generally used in mortality models, and the cohort modification of the LC model is one such example [14]. Herein, we assume that the parameters of the selection effect do not change with time.

First, we obtain the parameter estimates of the LC model (i.e. $\hat{\alpha}_x$,$\hat{\beta}_x$, and $\hat{\kappa}_t$) via the maximum likelihood estimation (MLE) method with a Poisson error structure and then estimate the selection effect (size [C_{xs} and lengths_x]). Note that we applied the "StMoMo" package in software R [18] for the parameter estimates of the LC model. Also note that $\sum_t \widehat{k}_t = 0$ and $\sum_x \widehat{\beta}_x = 1$ are the identifiability constraints used in "StMoMo" for the LC model. MLE estimates can also be obtained via other packages, such as "ilc" in R [3]. We only show the estimation results from the "StMoMo" package in this study, as the results from the "ilc" package are almost identical. If the model proposed in Eq. (2) is true, the difference between $\log(m_{xts})$ and $\hat{\alpha}_x + \hat{\beta}_x \hat{\kappa}_t$ can be used to estimate the selection effect.

Bear in mind that when we move from (t,s) to ($t+1$,$s+1$) after one year, a selection effect (s) and time effect (t) may be involved and confounded with each other. This study uses a two-stage estimation, and the selection and time effects were estimated separately. The initial estimates $\hat{\alpha}_x$,$\hat{\beta}_x$, and $\hat{\kappa}_t$ are typically biased, and a few iterations are required to stabilize the recursive estimation. We use the difference between $\log(m_{xts})$ and $C_{xs}I\{s \le s_x\}$ to revise the parameter estimates,$\hat{\alpha}_x$,$\hat{\beta}_x$, and $\hat{\kappa}_t$. In other words, a two-stage estimation was performed recursively until all parameter estimates converged. The convergence criterion was chosen when the difference in estimates between two consecutive iterations is smaller than a selected threshold, such as in 10^{-4} and 10^{-6}. In addition, we computed Monte Carlo confidence intervals for the parameter estimates ($\hat{\alpha}_x$,$\hat{\beta}_x$, and $\hat{\kappa}_t$) as a double-check. The estimation process is summarized as follows: Furthermore, the weights in the algorithm are exposures.

Step 0. Let the selection effect be zero, or $\hat{C}_{xs} = 0$, for all x and s.

Step 1. Let $\log(m_{xt})^*$ be the weighted average of $\log(m_{xts}) - \hat{C}_{xs}I\{s \le s_x\}$ over all policy years and apply the MLE to $\log(m_{xt})^*$ for estimates $\hat{\alpha}_x$,$\hat{\beta}_x$, and $\hat{\kappa}_t$.

Step 2. Compute the difference between $\log(m_{xts})$ and $\hat{\alpha}_x + \hat{\beta}_x \hat{\kappa}_t$ and define the residuals of the selection effect $e_{xts} = \log(m_{xts}) - \hat{\alpha}_x - \hat{\beta}_x \hat{\kappa}_t$. Next, let the estimate of the selection effect $\hat{C}_{xs}$ equal the weighted average of e_{xts} over all t.

Step 3. Repeat Steps 1 and 2 until all differences in the parameter estimates ($\hat{\alpha}_x$,$\hat{\beta}_x$,$\hat{\kappa}_t$, and $\hat{C}_{xs}$) between two consecutive iterations become smaller than a selected threshold.

In the following two sections, we use simulations and empirical data to evaluate the proposed model. First, we use the simulation to analyze the two-stage estimation process and verify whether it can provide unbiased and stable parameters and mortality rates. Since the preceding iteration process was simple and easy to use, it converged in a few seconds and fewer than 15 iterations. Then, we compare the parameter estimates between consecutive iterations and check whether these estimates converged.

Table 1 Size of selection effect (Simulation)

Ages	Policy Year									
	1	2	3	4	5	6	7	8	9	10+
15–19	0.7	0.8	0.9	1	1	1	1	1	1	1
20–24	0.7	0.8	0.9	1	1	1	1	1	1	1
25–29	0.6	0.7	0.8	0.9	1	1	1	1	1	1
30–34	0.6	0.7	0.8	0.9	1	1	1	1	1	1
35–39	0.5	0.6	0.7	0.8	0.9	1	1	1	1	1
40–44	0.5	0.6	0.7	0.8	0.9	1	1	1	1	1
45–49	0.4	0.5	0.6	0.7	0.8	0.9	1	1	1	1
50–54	0.4	0.5	0.6	0.7	0.8	0.9	1	1	1	1
55–59	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1	1
60–64	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1	1
65–69	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1

In addition to the proposed model in Eq. (2), we add a reduced model that includes only age and policy year effects.

$$\log(m_{\mathrm{xs}}) = \alpha^*_{\mathrm{x}} + \beta^*_{\mathrm{x}}\,\kappa^*_{\mathrm{s}} + \varepsilon^*_{\mathrm{xs}}, \tag{3}$$

where m_{xs} is the central mortality rate for age x and policy year s. The reduced model in Eq. (3) includes only the factor of the policy year, that is, year "t" in the LC model is replaced by policy year "s." To simplify the calculation, we assume that κ^*_s is linear in s for the rest of this study. However, parameter κ_t in LC-type models is non-linear, and we suggest conducting initial data analysis before applying a linear assumption to the reduced model.

The reduced model can be introduced because it can provide a rough estimate of the selection effect without many computations. It is especially useful when we have a shorter data period, and the mortality improvement scale is not significant. As we will see in the following two sections, we believe that the reduced model can be treated as an exploratory data analysis tool to check for a selection effect. However, we do not suggest using the reduced model to forecast mortality rates since it fails to consider mortality reduction over time, which is crucial in mortality models.

3 Simulation results

We evaluate the performance of the proposed approach using a simulation study and empirical analysis, as described in Sects. 3 and 4. First, we use a simulation to examine the estimation properties of the proposed approach in Sect. 3, followed by verifying whether the proposed models have smaller estimation errors than the LC model using empirical data in Sect. 4. In other words, the selection effect (Table 1) and mortality data (based on Taiwan's national population) are specified in Sect. 3. In Sect. 4, the proposed model is applied to real data from CLI.

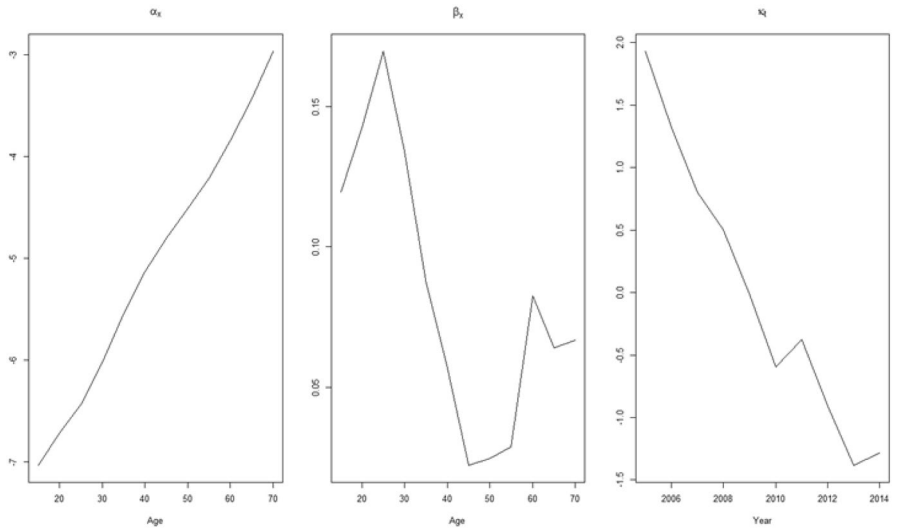

Fig. 1 Parameters of the LC Model (Data from Taiwan)

This section aims to mimic real-world situations. We generate artificial data according to some pre-specified assumptions and check whether the estimation results of the proposed approach meet our expectations. We need two sets of parameters: one for mortality reduction (i.e., the parameters of the LC model) and the other for the selection effect. The parameters of mortality reduction were obtained from the real experience of Taiwan's mortality data (2005–2014). We treated the estimates of α_x,β_x, and κ_t as true values (see Fig. 1 and Appendix). The parameters of the selection effect partly refer to the experienced values of CLI, with some modifications, making the selection effect size a smooth function of the policy year.

To demonstrate the proposed approach, we assume that the size of the selection period is a linear function of the policy year. Table 1 presents the values of the size of the selection effect for 12 age groups spanning five years, each for ten policy years. The mortality rates for each policy year are the mortality rates from the LC model multiplied by the values in Table 1. For example, a value of 0.8 for ages 15–19 at policy year 2 indicates that the mortality rate of this age group and policy year combination is 80% of the standard rate (i.e., 20% less) from the LC model. Moreover, the length of the selection effect was found to be a non-decreasing function of age, with a selection year of 3–8. Notably, the simulation setting suggests that the size of the selection effect is much larger than the mortality improvement. The 15–19 age group showed the largest annual mortality reduction (approximately 3%). However, this was smaller than the selection effect between the two policy years (e.g., policy years 1 and 2). Note that the size of the selection effect from the empirical analysis is a decreasing (but not necessarily linear) function of the policy year.

We only include the age groups of 15–19, …, 70–74, and omit ages below 15 and above 75, owing to the exposure sizes. Taiwan's regulation is another reason for excluding ages 0–14. The Taiwanese government established a rule for purchasing

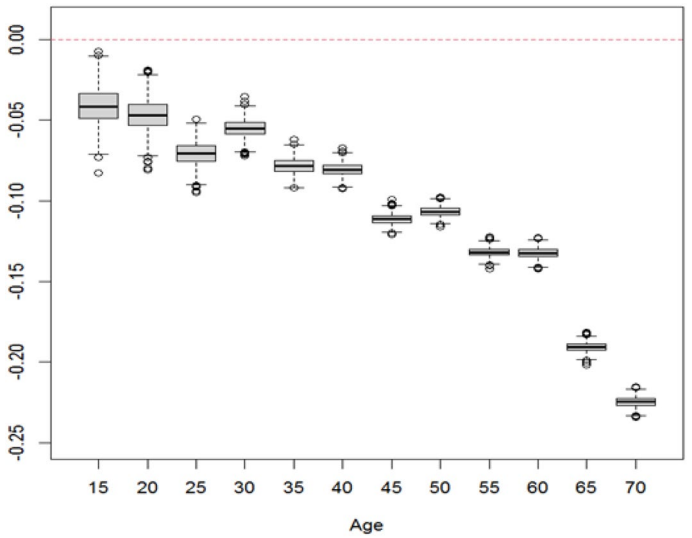

Fig. 2 Bias of α_x Estimate for the Full Model (1st Iteration)

life insurance products for the insured aged 0–14 in 2010 to avoid moral hazard from harming children. Insurance companies return the premiums paid and interest when the insured dies before 15 years. The data before and after 2010 may not be homogeneous; therefore, we did not consider mortality rates aged 0–14.

To allow artificial data to be as close as possible to the real case, we generated data according to Taiwan's populations (2005–2014). Approximately 23 million people live in Taiwan, and 2/3 have life insurance policies [22]. CLI is the largest life insurance company in Taiwan, and approximately eight million people hold life insurance policies with CLI. It was previously reported that the LC model yields unstable parameter estimates when the population size is small, especially when it is not larger than 200,000 [22]. Since the exposures from CLI are fairly large, parameter estimation of the LC model is expected to be suitable for this study. The estimates of α_x and β_x are likely to be biased if the exposures are small [21, 23]. We use the MLE method with a Poisson error structure to obtain parameter estimates of the LC model.[6] Additionally, we generate the number of deaths via Poisson distribution and then divide them into exposures to obtain the simulated mortality rates.

The simulation was repeated 10,000 times, and the estimates of all parameters were recorded. We first evaluate the full model in Eq. (2) and check if the estimates of parameters α_x, β_x, and κ_t are close to the true values. The estimates of α_x should be used as a demonstration (Figs. 2, 7 in Appendix). The estimates of β_x and κ_t are presented in Appendix. Figure 2 shows the bias of α_x estimate, that is, $E(\hat{\alpha}_x) - \alpha_x$, for the first iteration, and the estimates are negatively biased. The negative bias in α_x estimate is due to the LC model estimation process. The first step of the full model

[6] We also tried singular value decomposition for the parameter estimation, and the results were similar.

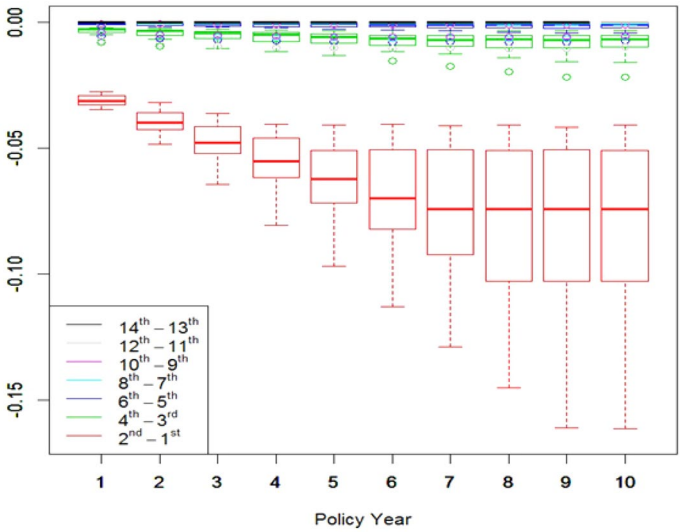

Fig. 3 Differences of Selection Effect Estimates between Two Iterations

estimation treats the average of $\log\left(m_{xts}\right)$ for all x and s as the α_x estimate. This process averages mortality rates overall policy durations, and the initial estimates of α_x are expected to be negatively biased.

The estimates of the parameters β_x and κ_t and the selection effect, are also biased for the first iteration. Note that the estimates of α_x, β_x, and κ_t at the first iteration were the same as those of the LC model, confirming that the mortality improvement and selection effects are confounded with each other. Conversely, the estimates of all the parameters improve with an increase in the number of iterations. We demonstrate the selection effect estimates for 15 iterations (Fig. 3), and observe that they converge quickly. Furthermore, the differences between the estimates of the two consecutive iterations are close to zero at the 15th iteration.

In addition to using the differences between the estimates of two consecutive iterations, we apply a Monte Carlo confidence interval to verify whether the parameter estimates converge to the true values. We choose 10^{-8} as the stopping criterion and find the estimates of all the parameters from the simulation satisfying this stopping criterion at the 15th iteration. Note that we used the floating-point format to store numbers on computers, and 10^{-8} was used as the basic unit (i.e., single precision) to record real numbers. Next, we compute the Monte Carlo confidence intervals of all parameters based on the estimates of the 15th iteration. The 2.5 and 97.5 percentiles of the parameter estimates are treated as the lower and upper bounds of 95% confidence intervals, respectively. The Monte Carlo confidence intervals cover the actual values of all the parameters (Appendix), indicating that the proposed estimation method is feasible and reliable. The lowest age groups have a relatively wider sampling distribution due to a few larger estimates, and there are approximately 1.5% of α_x estimates with bias exceeding 0.1. The difference between the average α_x estimates and the true parameters is approximately 0.01, less than 0.02% of the true α_x.

Table 2 Average estimates of the selection effect (15th Iteration, Full Model)

Ages	Policy Year									
	1	2	3	4	5	6	7	8	9	10+
15–19	0.6918	0.7925	0.8900	0.9851	0.9867	0.9883	0.9881	0.9898	0.9874	0.9888
20–24	0.6977	0.7979	0.8960	0.9952	0.9953	0.9964	0.9966	0.9959	0.9964	0.9962
25–29	0.5982	0.6997	0.8006	0.9000	0.9999	0.9991	0.9989	0.9989	0.9992	0.9988
30–34	0.6004	0.7001	0.7998	0.8992	1.0013	0.9994	1.0003	0.9981	1.0005	0.9997
35–39	0.4989	0.5986	0.7004	0.7995	0.9006	1.0002	0.9990	0.9989	1.0002	0.9999
40–44	0.5000	0.6004	0.6994	0.7998	0.9003	0.9989	0.9989	1.0004	1.0000	1.0001
45–49	0.4003	0.5001	0.6000	0.6996	0.7998	0.8999	10.0004	1.0000	0.9993	1.0000
50–54	0.4001	0.5007	0.6000	0.7002	0.7995	0.9001	10.0009	0.9995	1.0003	1.0000
55–59	0.2999	0.3997	0.4997	0.6000	0.7004	0.7995	0.8998	1.0001	0.9999	1.0000
60–64	0.2997	0.4001	0.4997	0.6003	0.7001	0.8010	0.8996	1.0001	1.0002	1.0000
65–69	0.1998	0.3003	0.4004	0.4999	0.6001	0.6995	0.8007	0.8997	0.9994	1.0000
70–74	0.2000	0.3000	0.4004	0.4999	0.6002	0.7006	0.8002	0.9005	1.0001	1.0000

Appendix also shows the bias of α_x,β_x, and κ_t estimates at the 1st and 15th iterations, and the convergence of β_x and κ_t estimates were much faster.

The bias of the selection effect estimates is minimal for all combinations of age and policy year at the 15th iteration, as shown in Table 2. Notably, the proposed estimation process provides stable estimates of the parameters for the full model in Eq. (2).[7] The evaluation of the reduced model in Eq. (3) was conducted similarly to the model in Eq. (2), and we only show the differences in the selection effect estimates between the reduced and full models (Fig. 4). Note that the differences are defined as the values of (full model-reduced model). The estimates from the reduced model are negatively biased (i.e., lower mortality estimates) for all combinations of age and policy year. The bias of the reduced model is especially noticeable in the higher age groups and middle policy years (e.g., policy years 5 and 6). Notably, the reduced model (i.e., without considering mortality reduction) would overestimate the mortality rates within and outside the selected period.

We compare the estimates of the lengths of the selected periods for the full and reduced models. Table 3 presents the actual and estimated lengths of the selected periods for the two proposed mortality models. The estimated lengths of the selected periods were obtained from the Monte Carlo p value depending on whether the estimated size of the selection effect was significantly different from 1. The full model provides accurate estimates of the length of the selected period. However, the reduced model underestimated the size and length of the

[7] The estimates of parameters α_x, β_x, κ_t, and C_{xs} are derived from the "StMoMo" package. The estimates obtained from the "ilc" package are extremely similar to those from "StMoMo." The α_x estimates from "ilc" are also negatively biased at the 1st iteration.

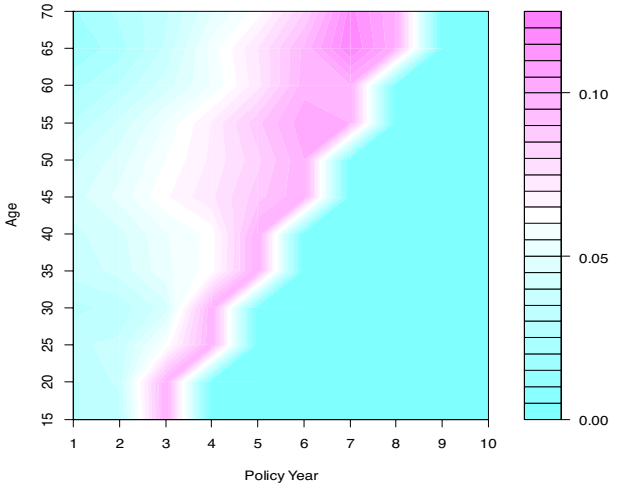

Fig. 4 Selection Effect Differences between Full and Reduced Models (Simulation)

study period. The selection effect was confounded by mortality reduction. Therefore, both factors should be included in the mortality rate model.

The mean absolute percentage error (MAPE) of mortality rates can also be used to evaluate the proposed model and is defined as

$$\text{MAPE} \;=\; \text{Average of}\left(\sum_{x,t,s}\left|\frac{\log(\hat{m}_{xts})-\log(m_{xts})}{\log(m_{xts})}\right|\right)\times 100\%. \tag{4}$$

There are two reasons for considering $\log(\hat{m}_{xts})$, instead of m_{xts} in Eq. (4). First, we estimate $\log\left(\hat{m}_{xts}\right)$ using the LC model, and it is natural to measure the model performance concerning the target variable. The other reason is that the range of m_{xts} is extremely large, ranging from 0.0005 to 0.05 for ages 15–74; taking the logarithm would narrow down the scale at different ages. Table 4 presents the MAPEs of the LC, full (Eq. 2), and reduced (Eq. 3) models. Again, the full model exhibits the smallest error. Surprisingly, the LC model fit well, although the true model was the LC model with the selection effect. The main reason for this behavior is that the sizes of the selection effect are linear functions of the policy year, and the selection effects are larger than the mortality reductions. This also explains why the MAPE of the reduced model is very small. Conversely, the MAPE of the reduced model is smaller than that of the LC model, and capturing the selection effect becomes more important than the mortality reduction trend in a shorter observation period (10 years in the simulation study). This suggests that the reduced model can be used as an exploratory data analysis tool to check for selection effects. It is possible to confuse the cohort effect with the selection effect. However, due to Taiwan's short data period, there is insufficient evidence of a cohort effect, and we ignore this possibility.

Table 3 Estimates of the length of the selection effect (Simulation)

Ages	15–19	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59	60–64	65–69	70–74
True	3	3	4	4	5	5	6	6	7	7	8	8
Full	3	3	4	4	5	5	6	6	7	7	8	8
Reduced	2	2	3	3	4	4	5	5	6	6	7	7

The cells on last line are the underestimated rates

Table 4 MAPE of mortality estimates (Simulation)

	LC model	Full model	Reduced model
MAPE	5.15%	0.03%	1.52%

Table 5 Various data features of CLI insureds

Age Structure										
Ages	15–19	20–24	25–29	30–34	35–39	40–44				
Proportion	7.5%	8.8%	11.2%	12.0%	10.8%	10.5%				
Ages	45–49	50–54	55–59	60–64	65–69	70–74				
Proportion	11.1%	10.8%	8.3%	4.8%	2.6%	1.6%				
Time Trend										
Calendar Year	2005	2006	2007	2008	2009					
Proportion	9.0%	9.1%	9.2%	9.3%	9.5%					
Calendar Year	2010	2011	2012	2013	2014					
Proportion	9.8%	10.3%	10.9%	11.3%	11.6%					
Policy Year										
Policy Year	1	2	3	4	5	6	7	8	9	10
Proportion	5.3%	5.0%	4.8%	4.5%	4.3%	4.3%	4.1%	4.5%	4.9%	5.1%
Policy Year	11	12	13	14	15	16	17	18	19	20+
Proportion	5.1%	5.3%	5.2%	5.3%	5.1%	4.8%	4.3%	3.7%	3.3%	11.0%

4 Empirical results

For the empirical study, we adopted the same age setting (12 age groups ranging from 5 years) but with a different policy year setting (20 policy years) for the study period of 2005–2014, based on the actual data (including age-specific exposures and numbers of deaths) from the CLI. These data are from whole-life and term-life policies for more than ten years. More women (51.7%) are insured than men (48.3%). A summary of the other data features (age, calendar year, and policy year) is presented in Table 5. On average, the CLI insured is 39.9 years old, with 10.9 policy years. Most insured individuals are between 25 and 29 and 50 and 54, and the total number of policies increases with time. For example, the number of policies in 2005 and 2014 is 9.0% and 11.6% of all policies, respectively, a significant increase (approximately 30% in 9 years). Moreover, there are many whole-life policies in Taiwan; thus, the proportion of 20 and more policy years (denoted by 20+) is 11%. Note that the claim system of the CLI was under major reorganization in the early 2000s, and the sales of life insurance policies increased significantly at the turn of the twenty-first century. We chose experienced data for the last ten years to avoid any data inhomogeneity problem.

We also used CLI data to evaluate the mortality rates fitting for the LC, full, and reduced models. To set the length of the selection effect, we chose a threshold

value of 0.95 as a threshold based on our simulation study. If the estimated values of the sizes of the selection effect are smaller (or larger) than 0.95, there are (or no) selection effects. Note that we can choose the time series or random walk assumption to quantify the confidence intervals [9, 24] in addition to the bootstrap simulation.

First, we compare the estimation results of the selection effect for the full and reduced models. The estimation results are similar to those of the simulation study. Generally, the reduced model shows shorter and smaller estimates of the selection effects. The estimated lengths of the selection effects are presented in Table 6. The full model also shows longer selection effects than the reduced model for all age groups. Similarly, the estimated sizes of the selection effect for the full model are larger than those of the reduced model, especially for the younger (ages 20–24 and 25–29) and middle-aged (around ages 50–64) age groups (Fig. 5). We note that the differences in Fig. 5 are the values of (full model-reduced model).

Additionally, the sizes of the selection effect are large; particularly, they were 50% larger for the elderly and early policy years (similar to those in Table 1). On average, the selection effect is larger than the smoker or non-smoker [15] and single or married effects [20]. Conversely, we believe that the selection effect might change with time as the improvement of technology in medical examination and big data analytics continues. However, we have only 10-year data (2005–2014). Additionally, we found that the selection effects of 2005–2009 and 2010–2014 show slight differences. We need to accumulate data for a longer period to ensure that the selection effect is constant over time.

Next, we compare the fitting performance of the three mortality models concerning the MAPE in the empirical study (Table 7). The full model has the smallest MAPE, and all three generated satisfactory estimation results. Although there are selection effects, the LC model is still a good mortality model to consider, probably because the trend in mortality improvement was consistent. Conversely, the empirical results suggest that the size of the selection effect is much larger than the mortality improvement, similar to the setting in the simulation study. Thus, the reduced model has a smaller MAPE than the LC model. We also compute Akaike information criterion (AIC) values for all the models to avoid using too many parameters, that is, $\mathrm{AIC} = 2\,\mathrm{k} - 2\log(\hat{L})$, where k is the number of parameters and $\hat{L}$ is the maximized value of the likelihood function for the model. The preferred model is the minimum AIC, and the full model again outperforms all the other models. Furthermore, the reduced model outperforms the LC model, similar to the simulation results. Moreover, including the selection effect appears to be more significant than including mortality reduction for a shorter observation period.

We further evaluate the models based on the mortality estimates of the CLI data for the year and the policy year (Fig. 6). We do not show the mortality estimates of the three models concerning age because they are extremely similar to the observed values. Although all three models have relatively small MAPEs, the mortality estimates are not the same as year and policy year. The reduced model has a significant bias with the year (left panel of Fig. 6), possibly due to the size of the selection effect (larger than the mortality improvement). Therefore, the reduced model should not be used in practice. Conversely, the LC model creates an apparent negative bias

Table 6 Estimates of the length of the selection effect (Empirical)

Ages	15–19	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59	60–64	65–69	70–74
Full	3	7	11	15	9	7	9	10	14	17	14	17
Reduced	1	3	4	4	4	5	6	6	6	8	8	7

The cells on last line are those with shorter select period

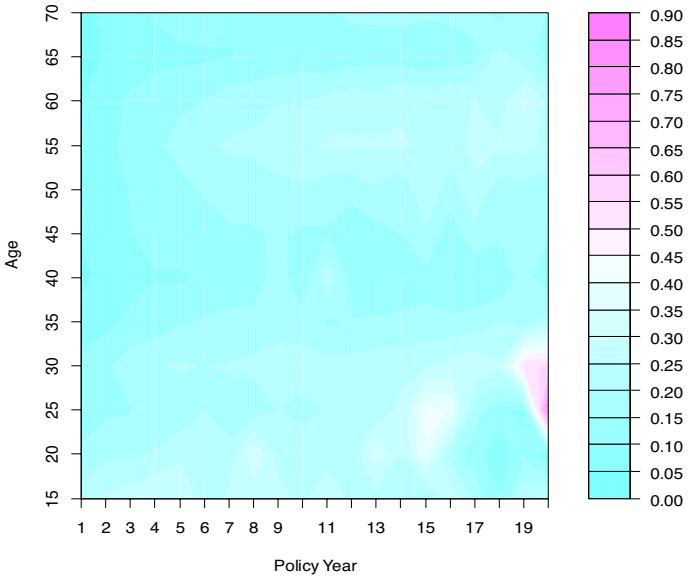

Fig. 5 Selection Effect Differences between Full and Reduced Models (Empirical)

Table 7 MAPE of mortality estimates (Empirical)

	LC model	Full model	Reduced model
MAPE	4.66%	2.64%	3.61%
AIC	− 29,254	− 33,292	− 31,486

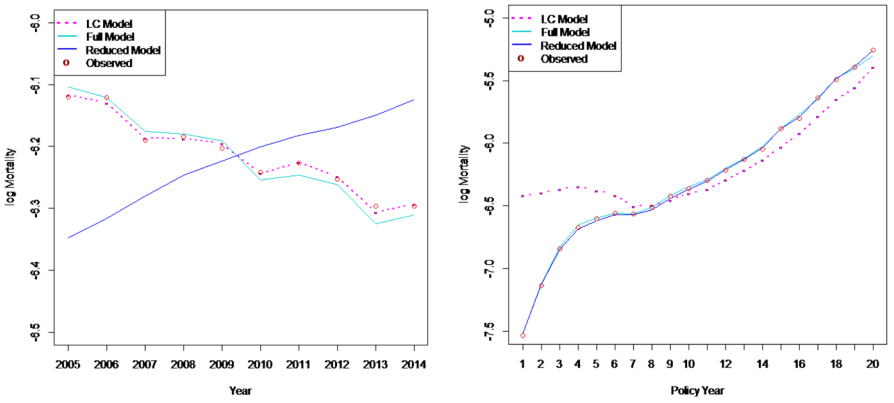

Fig. 6 Mortality Estimates with Respect to Year and Policy Year

concerning the policy year, ignoring the selection effect. Consequently, only the full model produces mortality estimates similar to the observed values, regardless of age, year, or policy year. Thus, it can be inferred that by including all three factors

(i.e., age, year, and policy year), it is possible to capture the mortality trend of the data from the CLI.

5 Conclusions

This study proposed a mortality model and an estimation method for mortality improvement and the selection effect. Simulation results confirm that the proposed approach and its estimation method provide stable and reliable estimates. Conversely, the regular LC model showed biased estimates of mortality reduction parameters. Without including the mortality improvement, the reduced model also produced biased estimates. The results of this empirical study also support the proposed approach. It appears that both the mortality improvement and selection effects exist and should be included in the mortality model. Moreover, the size of the selection, the effect is larger than that of the mortality reduction. Including the selection effect is more important than the trend of mortality reduction for a shorter observation period.

The estimated sizes of the selection effect are significant, that is, much larger than the effects of smoker or non-smoker and marriage status [20]. Therefore, they should be included in the pricing of life insurance products. Of course, lower mortality rates for early policy years lead to more flexibility and possibilities, such as reserve calculations and product design. For example, insurance companies can provide free or discounted health examinations to the insured as a bonus for mortality improvement every five or ten years. In this regard, if an insured shows a satisfactory condition from health examinations, they can receive a discount in the annual premium. Additionally, as lower mortality rates indicate lower risk to insurance companies, the insured and insurance companies can benefit from such modifications. Furthermore, this study can be generalized to other insurance companies in Taiwan (and other countries) to explore the nature of the selection effect. However, this depends on the support of insurance companies, such as whether there is sufficient exposure and the underwriting process is consistent over time, especially if they are willing to share their data. Unfortunately, Taiwanese insurance companies usually treat their experienced data as confidential data.

We believe that our findings on the selection effect are novel and that previous studies have not reported similar findings. However, we do not know the exact cause of the noticeable selection effect at older ages; insurable age might be a possible cause. In Taiwan, no insurance policies are usually offered to those older than 75 years (70 for some policies). Consequently, only 9% of the total life insurance exposures are aged 60 and over, indicating that insurance companies are more cautious in granting insurance policies to older people. Furthermore, it would require a long time and significant exposure to observe the trend in the selection effect. Therefore, we selected CLI as the study object. The CLI was established in 1962 and is the largest life insurer in Taiwan.

The proposed estimation comprises a two-stage iterative process. The estimation stabilized after the 4th iteration and converged before the 15th iteration (i.e., estimates between two consecutive iterations $|\hat{\theta}_{i+1} - \hat{\theta}_i| < 10^{-8}$ for $i \geq 15$). Interestingly,

the convergence rate was not the same for all parameters, and it was found that the estimates of β_x and κ_t were more sensitive. However, simulation and empirical studies have shown that this is not true. Although the role of parameters α_x and selection effects were similar to that of the intercept and dummy variables, they required more iteration steps to become stable, especially the selection effect. This contradicted our intuition but matched our research results on the LC model when the population size was small, where the estimate of parameter α_x was more sensitive to the population size [23]. The mortality rate within the selected period was a monotonic function of time; however, the proposed approach did not include this information in the model. We may use constrained estimation or the Lagrange multiplier method to restrict the range of estimates for the selection effect.

Although the mortality reduction and selection effect can be easily mixed, they do not result in linear dependency, not similar to the cohort modification of the LC model [14] and the Age-Period-Cohort model [20]. This is perhaps why the proposed approach's parameter estimates converged relatively quickly. If we consider more factors in the mortality model, we need to focus more on the estimation methods. We expect that the iteration process will be unstable if linear dependency or similar problems have existed. This situation can become even more challenging if the target population's exposure (or population size) is small [21]. We may need to employ techniques in variance reduction, such as importance sampling and control variates, to acquire stable parameter estimates.

Appendix: Estimates and Monte Carlo Confidence Intervals of α_x, β_x, and κ_t (Simulation)

Bias of α_x Estimates

See Fig. 7

Bias of β_x Estimates

See Fig. 8

Bias of κ_t Estimates

See Fig. 9

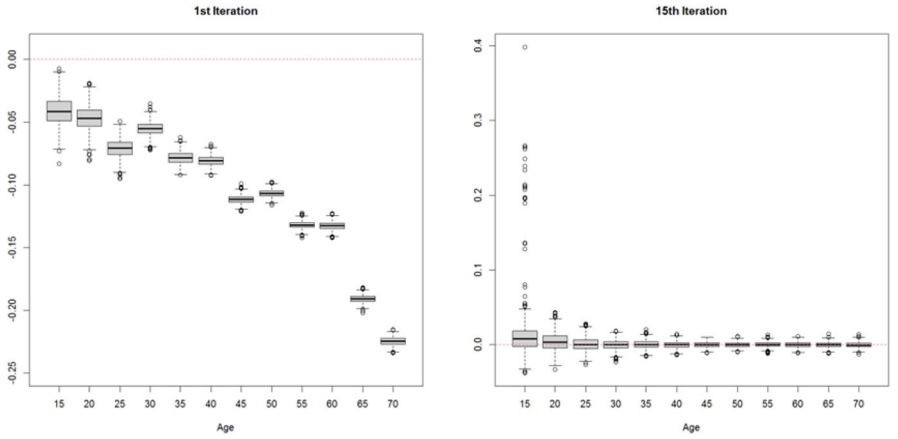

Fig. 7 Bias of Estimates for Parameter α_x(1st and 15th iteration)

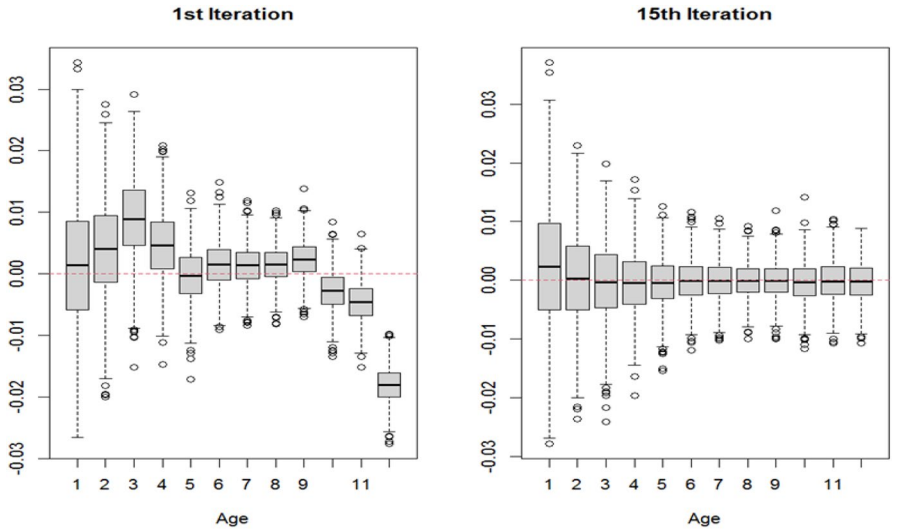

Fig. 8 Bias of Estimates for Parameter β_x (1st and 15th iteration)

Estimates of Selection Effect

See Fig. 10

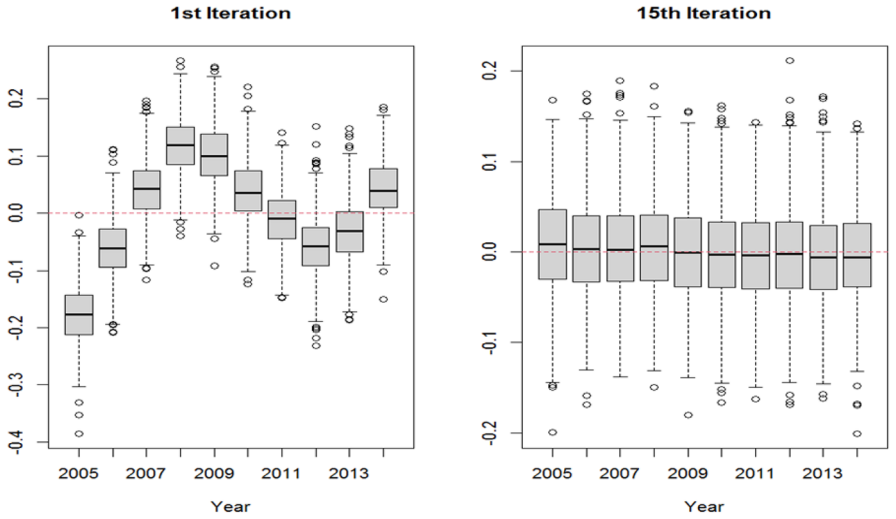

Fig. 9 Bias of Estimates for Parameter κ_t(1st and 15th iteration)

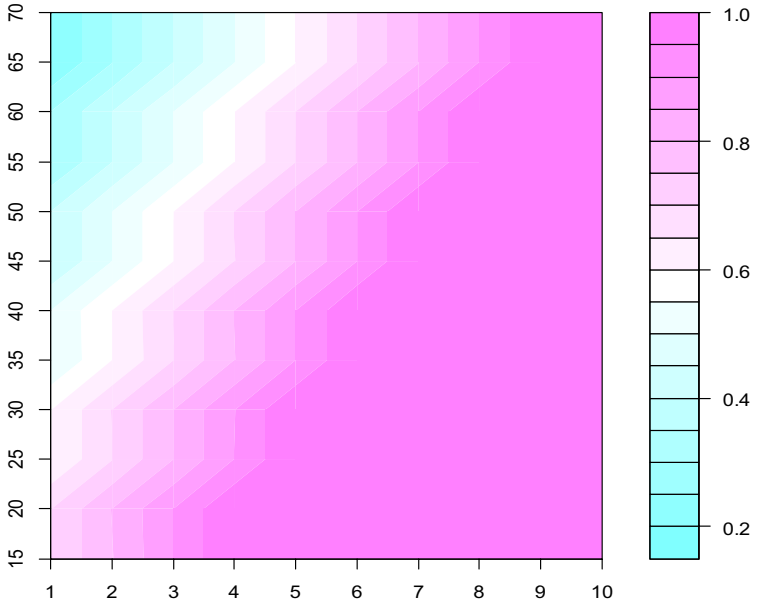

Fig. 10 Estimates of Selection Effect (15th iteration)

Parameters α_x,β_x, and κ_t with the Monte Carlo 95% Confidence Intervals

See Tables 8 and 9

Table 8 Parameters α_x and β_x with their Monte Carlo 95% Confidence Intervals

Ages	α_x			β_x		
	True	Lower	Upper	True	Lower	Upper
15–19	− 7.0373	− 7.0586	− 6.9893	0.119549	0.101459	0.141123
20–24	− 6.7161	− 6.7356	− 6.6895	0.142789	0.126561	0.156717
25–29	− 6.4327	− 6.4491	− 6.4129	0.169877	0.156357	0.182158
30–34	− 6.0174	− 6.0309	− 6.0051	0.134303	0.123443	0.144662
35–39	− 5.5471	− 5.5576	− 5.5358	0.088160	0.079217	0.095998
40–44	− 5.1332	− 5.1416	− 5.1246	0.056840	0.050118	0.063495
45–49	− 4.8009	− 4.8079	− 4.7938	0.022028	0.015211	0.028349
50–54	− 4.5123	− 4.5190	− 4.5059	0.024510	0.018643	0.030241
55–59	− 4.2124	− 4.2190	− 4.2053	0.028670	0.022625	0.034549
60–64	− 3.8476	− 3.8542	− 3.8403	0.082479	0.075641	0.088487
65–69	− 3.4272	− 3.4339	− 3.4199	0.064053	0.057579	0.071064
70–74	− 2.9646	− 2.9719	− 2.9574	0.066741	0.059955	0.072729

Table 9 Parameters κ_t with their Monte Carlo 95% Confidence Intervals

Year	κ_t		
	True	Lower	Upper
2005	1.932799	1.835162	2.052252
2006	1.323163	1.225568	1.429404
2007	0.804807	0.710995	0.915893
2008	0.503682	0.399526	0.611485
2009	− 0.018732	− 0.126364	0.076648
2010	− 0.593321	− 0.703687	− 0.491194
2011	− 0.379154	− 0.487906	− 0.282482
2012	− 0.904560	− 1.010008	− 0.804055
2013	− 1.385099	− 1.492573	− 1.288221
2014	− 1.283585	− 1.388874	− 1.190643

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Availability of data and material The empirical data are the property of the Cathay Life Insurance Co. and are not open to the public.

Code availability The code is open source and is written in R statistical software.

Declarations

Conflict of interest We declare that we have no competing interests.

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