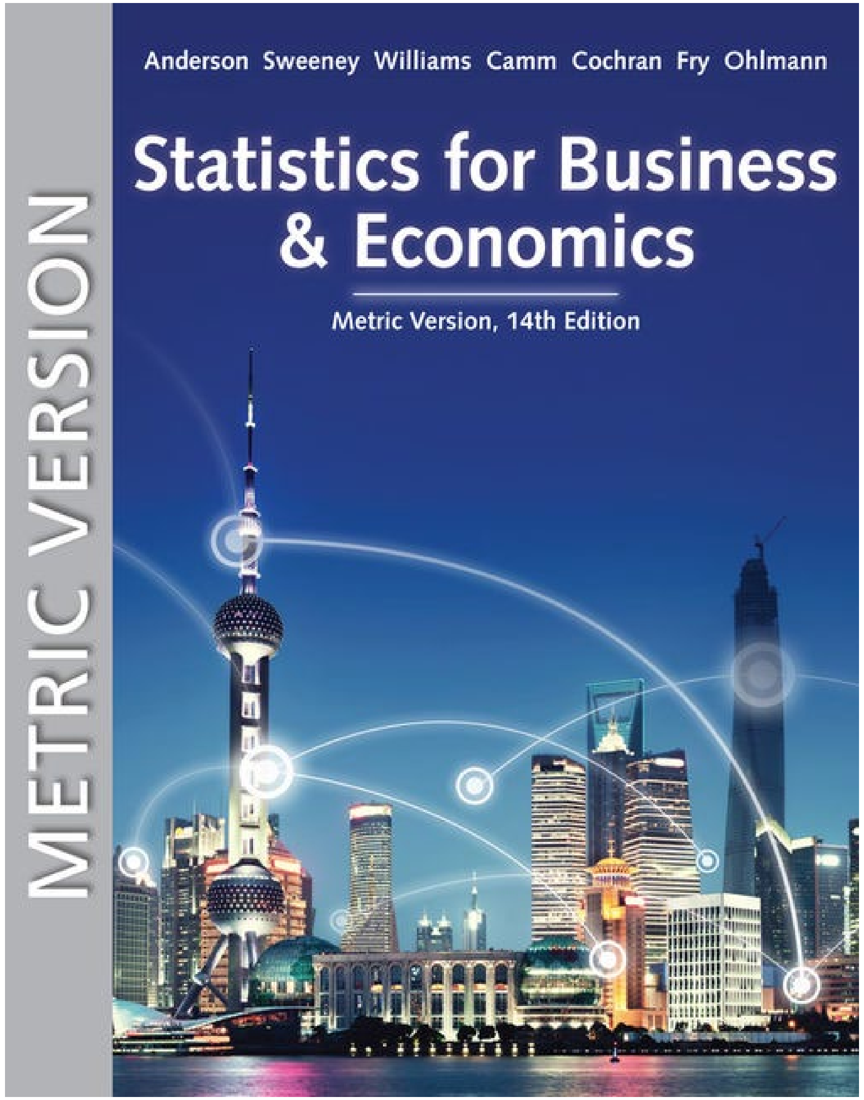
Statistics for Business and Economics (14e) Metric Version

Chapter 18 （無母數方法）



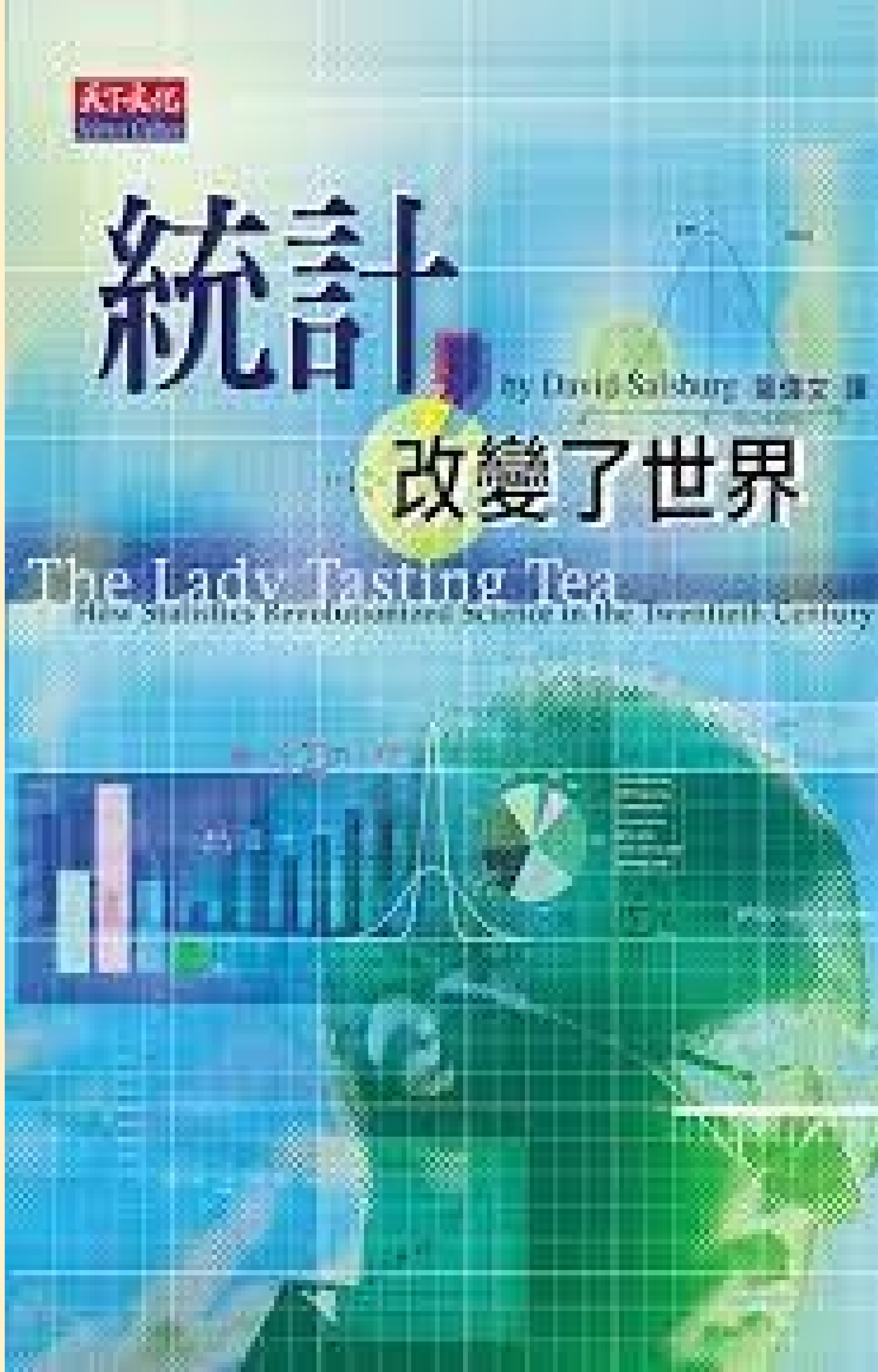
Chapter 18 - Nonparametric Methods

「淑女與下午茶」

- 在英國劍橋有位女士，聲稱把茶加到牛奶、把牛奶加到茶裡，兩種方法調出來的下午茶喝起來味道不同。在座的科學家都對她的說法嗤之以鼻，但有位來訪的瘦小紳士（RA費雪），提議要用科學的方法檢驗這位女士的假設。

→實驗設計(Experimental Design)

→如果十次測試，這位女士全部猜中茶加奶、奶加茶的順序，可能會是亂猜嗎？



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【2022年2月14日】台南白河木屐寮舉辦擲筊大賽，只要有點燈，擲出兩個聖筊，就有機會參加比賽，最後由來自高雄的陳姓男子跟朋友，抱走66萬6666元大獎！

Nonparametric Methods

- Most of the statistical methods referred to as parametric require the use of interval-or ratio-scaled data.
- Nonparametric methods are often the only way to analyze categorical (nominal or ordinal) data and draw statistical conclusions.
- Nonparametric methods require no assumptions about the population probability distributions.
- Nonparametric methods are often called distribution-free methods.
- Whenever the data are quantitative, we will transform the data into categorical data in order to conduct the nonparametric test.

Sign Test

- The sign test is a versatile method for hypothesis testing that uses the binomial distribution with $p = .50$ as the sampling distribution.
- We present two applications of the sign test:
  - A hypothesis test about a population median
  - A matched-sample test about the difference between two populations

Hypothesis Test about a Population Median (1 of 2)

We can apply the sign test by:

- Using a <u>plus sign</u> whenever the data in the sample are above the hypothesized value of the median
- Using a <u>minus sign</u> whenever the data in the sample are below the hypothesized value of the median
- Discarding any data exactly equal to the hypothesized median

Hypothesis Test about a Population Median (2 of 2)

- The assigning of the plus and minus signs makes the situation into a binomial distribution application.
- The sample size is the number of trials.
- There are two outcomes possible per trial: a plus sign or a minus sign.
- The trials are independent.
- We let p denote the probability of a plus sign.
- If the population median is in fact a particular value, p should equal 0.5.

Hypothesis Test about a Population Median: Small-Sample Case

(1 of 6)

- The small-sample case for this sign test should be used whenever $n \leq 20$.
- The hypotheses are

H_0: $p = 0.50$	The population median equals the value assumed.
H_a: $p \neq 0.50$	The population median is different than the value assumed.

- The number of plus signs is our test statistic.
- Assuming H_0 is true, the sampling distribution for the test statistic is a binomial distribution with p = 0.5.
- H_0 is rejected if the p-value $\leq$ level of significance, α.

Hypothesis Test about a Population Median: Small-Sample Case

(2 of 6)

Lawler's Grocery Store made the decision to carry Cape May Potato Chips based on the manufacturer's estimate that the median sales should be $450 per week on a per-store basis.

Lawler's has been carrying the potato chips for three months. Data showing one-week sales at 10 randomly selected Lawler's stores are shown here.

Store Number	Weekly Sales	Sign	Store Number	Weekly Sales	Sign
56	$485	+	63	$474	+
19	562	+	39	662	+
36	415	-	84	380	-
128	860	+	102	515	+
12	426	-	44	721	+

Hypothesis Test about a Population Median: Small-Sample Case

(3 of 6)

Lawler's management requested the following hypothesis test about the population median weekly sales of Cape May Potato Chips (using $\alpha = 0.10$).

H_0: Median Sales = \$450

H_a: Median Sales ≠ \$450

In terms of the binomial probability p:

$H_0: p = 0.50$

$H_a: p \neq 0.50$

Hypothesis Test about a Population Median: Small-Sample Case

(4 of 6)

Example: Potato Chip Sales

Binomial Probabilities with n = 10 and p = 0.50

Number of Plus Signs	Probability	Number of Plus Signs	Probability
0	.0010	6	.2051
1	.0098	7	.1172
2	.0439	8	.0439
3	.1172	9	.0098
4	.2051	10	.0010
5	.2461		

Hypothesis Test about a Population Median: Small-Sample Case

(5 of 6)

Example: Potato Chip Sales

Because the observed number of plus signs is 7, we begin by computing the probability of obtaining 7 or more plus signs.

The probability of 7, 8, 9, or 10 plus signs is:

0.1172 + 0.0439 + 0.0098 + 0.0010 = 0.1719.

We are using a two-tailed hypothesis test, so:

p-value = 2(0.1719) = 0.3438.

With p-value > α, (0.3438 > 0.10), we cannot reject H_0.

Hypothesis Test about a Population Median: Small-Sample Case

(6 of 6)

Conclusion

Because the p-value > α, we cannot reject H_0. There is insufficient evidence in the sample to reject the assumption that the median weekly sales is $450.

Hypothesis Test about a Population Median: Larger Sample Size

(1 of 5)

With larger sample sizes, we rely on the normal distribution approximation of the binomial distribution to compute the p-value, which makes the computations quicker and easier.

Normal Approximation of the Number of Plus Signs when

$$H_0: p = 0.50$$

Mean: $\mu = 0.5n$

Standard Deviation: $\sigma = \sqrt{.25n}$

Distribution Form: Approximately normal for $n > 20$

Hypothesis Test about a Population Median: Larger Sample Size

Example: Trim Fitness Center

A hypothesis test is being conducted about the median age of female members of the Trim Fitness Center.

$$H_0\text{: Median Age} = 34 \text{ years}$$

$$H_a\text{: Median Age} \neq 34 \text{ years}$$

In a sample of 40 female members, 25 are older than 34, 14 are younger than 34, and 1 is 34. Is there sufficient evidence to reject H_0? Use $\alpha = 0.05$.

Hypothesis Test about a Population Median: Larger Sample Size

(3 of 5)

Example: Trim Fitness Center

- Letting x denote the number of plus signs, we will use the normal distribution to approximate the binomial probability $P(x \leq 25)$.
- Remember that the binomial distribution is discrete and the normal distribution is continuous.
- To account for this, the binomial probability of 25 is computed by the normal probability interval 24.5 to 25.5.

Hypothesis Test about a Population Median: Larger Sample Size

(4 of 5)

- Mean and Standard Deviation

$$\mu = 0.5n = 0.5(39) = 19.5$$

$$\sigma = \sqrt{.25n} = \sqrt{.25(39)} = 3.1225$$

- Test Statistic

$$z = \frac{x - \mu}{s} = \frac{24.5 - 19.5}{3.1225} = 1.6013$$

- p-value

p-value = 2(1 – 0.9453) = 0.1093

Hypothesis Test about a Population Median: Larger Sample Size

(5 of 5)

- Rejection Rule

Using 0.05 level of significance: Reject H_0 if p-value $\leq$ 0.05

- Conclusion

Do not reject H_0. The p-value for this two-tail test is .1093. There is insufficient evidence in the sample to conclude that the median age is not 34 for female members of Trim Fitness Center.

Hypothesis Test with Matched Samples

- A common application of the sign test involves using a sample of n potential customers to identify a preference for one of two brands of a product.
- The objective is to determine whether there is a difference in preference between the two items being compared.
- To record the preference data, we use a plus sign if the individual prefers one brand and a minus sign if the individual prefers the other brand.
- Because the data are recorded as plus and minus signs, this test is called the sign test.

Hypothesis Test with Matched Samples: Small-Sample Case (1 of 7)

- The small-sample case for the sign test should be used whenever $n \leq 20$.
- The hypotheses are

$H_0\colon p = 0.50$ No preference for one brand over the other exists.

$H_a\colon p \neq 0.50$ A preference for one brand over the other exists.

- The number of plus signs is our test statistic.
- Assuming H_0 is true, the sampling distribution for the test statistic is a binomial distribution with p = 0.5.
- H_0 is rejected if the p-value $\leq$ level of significance, α.

Hypothesis Test with Matched Samples: Small-Sample Case (2 of 7)

Example: Major Call Center

Maria Gonzales is the supervisor responsible for scheduling telephone operators at a major call center. She is interested in determining whether her operators' preferences between the day shift (7 a.m. to 3 p.m.) and evening shift (3 p.m. to 11 p.m.) are different.

Maria randomly selected a sample of 16 operators who were asked to state a preference for the one of the two work shifts. The data collected from the sample are shown on the next slide.

Hypothesis Test with Matched Samples: Small-Sample Case (3 of 7)

Example: Major Call Center

Worker	Shift Preference	Sign	Worker	Shift Preference	Sign
1	Day	+	9	Evening	-
2	Evening	-	10	Evening	-
3	Evening	-	11	Evening	-
4	Evening	-	12	(none)	
5	Day	+	13	Evening	-
6	Evening	-	14	Day	+
7	Day	+	15	Evening	-
8	(none)		16	Evening	-

Hypothesis Test with Matched Samples: Small-Sample Case (4 of 7)

Example: Major Call Center

4 plus signs

10 negative signs

(n = 14)

Can Maria conclude, using a level of significance of α = 0.10, that operator preferences are different for the two shifts?

$H_0\colon p = 0.50$ A preference for one shift over the other does exist.

$H_a\colon p \neq 0.50$ A preference for one shift over the other does not exist.

Hypothesis Test with Matched Samples: Small-Sample Case (5 of 7)

Example: Major Call Center

Binomial Probabilities with n = 14 and p = 0.50

Number of Plus Signs	Probability	Number of Plus Signs	Probability
0	.00006	8	.18329
1	.00085	9	.12219
2	.00555	10	.06110
3	.02222	11	.02222
4	.06110	12	.00555
5	.12219	13	.00085
6	.18329	14	.00006
7	.20947		

Hypothesis Test with Matched Samples: Small-Sample Case (6 of 7)

Example: Major Call Center

Because the observed number of plus signs is 4, we begin by computing the probability of obtaining 4 or less plus signs.

The probability of 0, 1, 2, 3, or 4 plus signs is:

$$.00006 + .00085 + .00555 + .02222 + .06110 = .08978.$$

- We are using a two-tailed hypothesis test, so the p-value = 2(0.08978) = 0.17956.
- With p-value > α, (0.17956 > 0.10), we cannot reject H_0.

Hypothesis Test with Matched Samples: Small-Sample Case (7 of 7)

Conclusion

Because the p-value > α, we cannot reject H_0. There is insufficient evidence in the sample to conclude that a difference in preference exists for the two work shifts.

Hypothesis Test with Matched Samples: Large-Sample Case (1 of 7)

- Using $H_0: p = 0.5$ and $n > 20$, the sampling distribution for the number of plus signs can be approximated by a normal distribution.
- When no preference is stated ($H_0: p = 0.5$), the sampling distribution will have:
- Mean: $\mu = 0.50n$
- Standard Deviation: $\sigma = \sqrt{.25n}$
- The test statistic is:
- $z = \frac{x-\mu}{\sigma}$ (x is the number of plus signs)
- H_0 is rejected if the p-value $\leq$ level of significance, α.

Hypothesis Test with Matched Samples: Large-Sample Case (2 of 7)

Example: Ketchup Taste Test

As part of a market research study, a sample of 80 consumers were asked to taste two brands of ketchup and indicate a preference. Do the data shown on the next slide indicate a significant difference in the consumer preferences for the two brands?

Hypothesis Test with Matched Samples: Large-Sample Case (3 of 7)

Example: Ketchup Taste Test

45	preferred Brand A Ketchup	(+ sign recorded)
27	preferred Brand B Ketchup	(– sign recorded)
8	had no preference	

The analysis will be based on a sample size of 45 + 27 = 72.

Hypothesis Test with Matched Samples: Large-Sample Case (4 of 7)

- Hypotheses

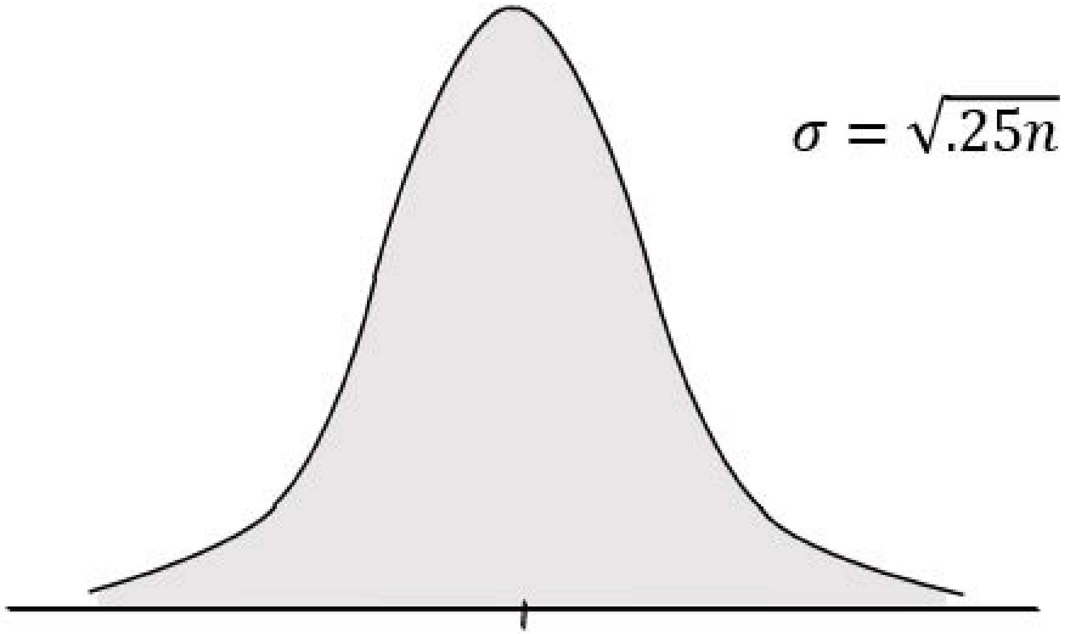
H_0: $p = .50$ (No preference for one brand over the other exists)

H_a: $p \neq .50$ (A preference for one brand over the other exists)

Hypothesis Test with Matched Samples: Large-Sample Case (5 of 7)

Sampling Distribution for Number of Plus Signs

$$\sigma = \sqrt{.25n} = \sqrt{.25(72)} = 4.243$$



$$\mu = .5(72) = 36$$

Hypothesis Test with Matched Samples: Large-Sample Case (6 of 7)

- Rejection Rule

Using 0.05 level of significance:

Reject H_0 if p-value $\leq$ 0.05

- Test Statistic

$$z = \frac{x - \mu}{\sigma} = \frac{44.5 - 36}{4.243} = 2.00$$

- p-value

p-value = 2(1 – 0.9875) = 0.045

Hypothesis Test with Matched Samples: Large-Sample Case (7 of 7)

- Conclusion

Because the p-value < α, we can reject H_0. There is sufficient evidence in the sample to conclude that a difference in preference exists for the two brands of ketchup.

Wilcoxon Signed-Rank Test (1 of 10)

- The Wilcoxon signed-rank test is a procedure for analyzing data from a matched samples experiment.
- The test uses quantitative data but does not require the assumption that the differences between the paired observations are normally distributed.
- It only requires the assumption that the differences have a symmetric distribution.
- This occurs whenever the shapes of the two populations are the same and the focus is on determining if there is a difference between the two populations’ medians.

Wilcoxon Signed-Rank Test (2 of 10)

- Let T^- denote the sum of the negative signed ranks.
- Let T^+ denote the sum of the positive signed ranks.
- If the medians of the two populations are equal, we would expect the sum of the negative signed ranks and the sum of the positive signed ranks to be approximately the same.
- We use T^+ as the test statistic.

Wilcoxon Signed-Rank Test (3 of 10)

Sampling Distribution of T^+ for the Wilcoxon Signed-Rank Test

Mean:

$$\mu_{T^+} = \frac{n(n+1)}{4}$$

Standard Deviation:

$$\sigma_{T^+} = \sqrt{\frac{n(n+1)(2n+1)}{24}}$$

Distribution Form:

Approximately normal for $n \geq 10$

Wilcoxon Signed-Rank Test (4 of 10)

Example: Express Deliveries

A firm has decided to select one of two express delivery services to provide next-day deliveries to its district offices.

To test the delivery times of the two services, the firm sends two reports to a sample of 10 district offices, with one report carried by one service and the other report carried by the second service. Do the data on the next slide indicate a difference in the two services?

Wilcoxon Signed-Rank Test (5 of 10)

District Office	OverNight	NiteFlight
Seattle	32 hrs.	25 hrs.
Los Angeles	30	24
Boston	19	15
Cleveland	16	15
New York	15	13
Houston	18	15
Atlanta	14	15
St. Louis	10	8
Milwaukee	7	9
Denver	16	11

Wilcoxon Signed-Rank Test (6 of 10)

Hypotheses

H_0: The difference in the median delivery times of the two services equals 0.

H_a: The difference in the median delivery times of the two services does not equal 0.

Wilcoxon Signed-Rank Test (7 of 10)

Preliminary Steps of the Test

- Compute the differences between the paired observations.
- Discard any differences of zero.
- Rank the absolute value of the differences from lowest to highest. Tied differences are assigned the average ranking of their positions.
- Give the ranks the sign of the original difference in the data.
- Sum the signed ranks.

. . . next we will determine whether the sum is significantly different from zero.

Wilcoxon Signed-Rank Test (8 of 10)

District Office	Differ.	|Diff.| Rank	Sign. Rank
Seattle	7	10	+10
Los Angeles	6	9	+9
Boston	4	7	+7
Cleveland	1	1.5	+1.5
New York	2	4	+4
Houston	3	6	+6
Atlanta	-1	1.5	-1.5
St. Louis	2	4	+4
Milwaukee	-2	4	-4
Denver	5	8	+8
			$T^+ = 49.5$

Wilcoxon Signed-Rank Test (9 of 10)

- Test Statistic

$$\mu_{T^+} = \frac{n(n+1)}{4} = \frac{10(10+1)}{4} = 27.5$$

$$\sigma_{T^+} = \sqrt{\frac{n(n+1)(2n+1)}{24}} = \sqrt{\frac{10(11)(21)}{24}} = 9.81$$

$$\mathrm{P}(T^+ \geq 49.5) = P\left[z \geq \frac{49.5 - 27.5}{9.81}\right] = P(z \geq 2.24)$$

- *p*-value

$$\text{p-value} = 2\,(1–0.9875) = 0.025$$

Wilcoxon Signed-Rank Test (10 of 10)

- Rejection Rule:

Using 0.05 level of significance

Reject H_0 if p-value ≤ 0.05

- Conclusion:

Reject H_0.

The p-value for this two-tail test is 0.025. There is sufficient evidence in the sample to conclude that a difference exists in the median delivery times provided by the two services.

Mann-Whitney-Wilcoxon Test (1 of 5)

- This test is another nonparametric method for determining whether there is a difference between two populations.
- This test is based on two independent samples.
- Advantages of this procedure:
  - It can be used with either ordinal data or quantitative data.
  - It does not require the assumption that the populations have a normal distribution.

Mann-Whitney-Wilcoxon Test (2 of 5)

Instead of testing for the difference between the medians of two populations, this method tests to determine whether the two populations are identical.

The hypotheses are:

H_0: The two populations are identical

H_a: The two populations are not identical

Mann-Whitney-Wilcoxon Test (3 of 5)

Example: Westin Freezers

Manufacturer labels indicate the annual energy cost associated with operating home appliances such as freezers.

The energy costs for a sample of 10 Westin freezers and a sample of 10 Easton Freezers are shown on the next slide. Do the data indicate, using $\alpha = 0.05$, that a difference exists in the annual energy costs for the two brands of freezers?

Mann-Whitney-Wilcoxon Test (4 of 5)

Westin Freezers	Easton Freezers
$55.10	$56.10
54.50	54.70
53.20	54.40
53.00	55.40
55.50	54.10
54.90	56.00
55.80	55.50
54.00	55.00
54.20	54.30
55.20	57.00

Mann-Whitney-Wilcoxon Test (5 of 5)

Hypotheses

H_0: Annual energy costs for Westin freezers and Easton freezers are the same.

H_a: Annual energy costs differ for the two brands of freezers.

Mann-Whitney-Wilcoxon Test: Large-Sample Case (1 of 2)

- First, rank the combined data from the lowest to the highest values, with tied values being assigned the average of the tied rankings.
- Then, compute W, the sum of the ranks for the first sample.
- Then, compare the observed value of W to the sampling distribution of W for identical populations. The value of the standardized test statistic z will provide the basis for deciding whether to reject H_0.

Mann-Whitney-Wilcoxon Test: Large-Sample Case (2of 2)

Sampling Distribution of W with Identical Populations

- Mean

$$m_W = \frac{n_1(n_1 + n_2 + 1)}{2}$$

- Standard Deviation

$$\sigma_W = \sqrt{{}^{1}/_{12}\, n_1 n_2 (n_1 + n_2 + 1)}$$

- Distribution Form

Approximately normal, provided $n_1 \geq 7$ and $n_2 \geq 7$

Mann-Whitney-Wilcoxon Test (1 of 4)

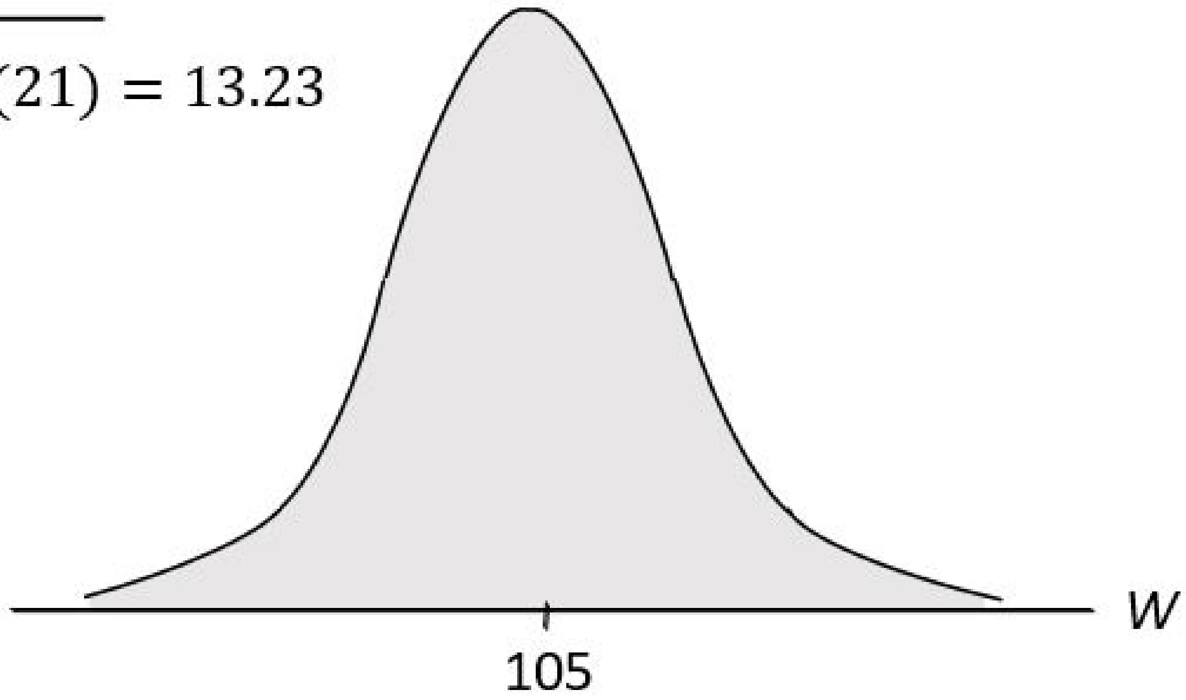
Westin Freezers	Rank	Easton Freezers	Rank
$55.10	12	$56.10	19
54.50	8	54.70	9
53.20	2	54.40	7
53.00	1	55.40	14
55.50	15.5	54.10	4
54.90	10	56.00	18
55.80	17	55.50	15.5
54.00	3	55.00	11
54.20	5	54.30	6
55.20	13	57.00	20
Sum of Ranks	86.5	Sum of Ranks	123.5

Mann-Whitney-Wilcoxon Test (2 of 4)

Sampling Distribution of W with Identical Populations

$$\sigma_W = \sqrt{{}^{1}/_{12}(10)(10)(21)} = 13.23$$

$$\mu_W = ½(10)(21) = 105$$



Mann-Whitney-Wilcoxon Test (3 of 4)

- Rejection Rule

Using 0.05 level of significance,

Reject H_0 if p-value $\leq$ 0.05

- Test Statistic

$$P(W \leq 86.5) = P\left[z \leq \frac{86.5-105}{13.23}\right] = P(z \leq -1.40)$$

- p-value

p-value = 2(0.0808) = 0.1616

Mann-Whitney-Wilcoxon Test (4 of 4)

Conclusion

Do not reject H_0. The p-value > α. There is insufficient evidence in the sample data to conclude that there is a difference in the annual energy cost associated with the two brands of freezers.

Kruskal-Wallis Test (1 of 8)

The Mann-Whitney-Wilcoxon test has been extended by Kruskal and Wallis for cases of three or more populations.

$$H_0\text{: All populations are identical}$$

$$H_a\text{: Not all populations are identical}$$

- The Kruskal-Wallis test can be used with ordinal data as well as with interval or ratio data.
- Also, the Kruskal-Wallis test does not require the assumption of normally distributed populations.

Kruskal-Wallis Test (2 of 8)

Test Statistic

$$H = \left[\frac{12}{n_T(n_T+1)}\sum_{i=1}^{k}\frac{R_i^{\,2}}{n_i}\right] - 3(n_T+1)$$

where:

k = number of populations

n_i = number of observations in sample i

n_T = Sn_i = total number of observations in all samples

R_i = sum of the ranks for sample i

Kruskal-Wallis Test (3 of 8)

- When the populations are identical, the sampling distribution of the test statistic H can be approximated by a chi-square distribution with $k - 1$ degrees of freedom.
- This approximation is acceptable if each of the sample sizes $n_i \geq 5$.
- This test is always expressed as an upper-tailed test.
- The rejection rule is: Reject H_0 if p-value $\leq \alpha$.

Kruskal-Wallis Test (4 of 8)

Example: Lakewood High School

John Norr, Director of Athletics at Lakewood High School, is curious about whether a student’s total number of absences in four years of high school is the same for students participating in no varsity sport, one varsity sport, and two varsity sports.

Number of absences data were available for 20 recent graduates and are listed on the next slide. Test whether the three populations are identical in terms of number of absences. Use $\alpha = 0.10$.

Kruskal-Wallis Test (5 of 8)

Example: Lakewood High School

No Sport	1 Sport	2 Sports
13	18	12
16	12	22
6	19	9
27	7	11
20	15	15
14	20	21
	17	10

Kruskal-Wallis Test (6 of 8)

Example: Lakewood High School

No Sport	Rank	1 Sport	Rank	2 Sports	Rank
13	8	18	14	12	6.5
16	12	12	6.5	22	19
6	1	19	15	9	3
27	20	7	2	11	5
20	16.5	15	10.5	15	10.5
14	9	20	16.5	21	18
		17	13	10	4
Total	66.5		77.5		66

Kruskal-Wallis Test (7 of 8)

Rejection Rule

Using test statistic: Reject H_0 if $\chi^2 \geq 4.60517$ (2 *df*)

Using p-value: Reject H_0 if p-value ≤ 0.10

Kruskal-Wallis Test Statistic

k = 3 populations, n_1 = 6, n_2 = 7, n_3 = 7, n_T = 20

$$H = \left[\frac{12}{n_T(n_T+1)}\sum_{i=1}^{k}\frac{{R_i}^2}{n_i}\right] - 3(n_T+1)$$

$$H = \left[\frac{12}{20(20+1)}\left[\frac{(66.5)^2}{6} + \frac{(77.5)^2}{7} + \frac{(66.0)^2}{7}\right]\right] - 3(20+1) = 0.3532$$

Kruskal-Wallis Test (8 of 8)

Conclusion

Do no reject H_0. There is insufficient evidence to conclude that the populations are not identical. (H = 0.3532 < 4.60517)

Rank Correlation (1 of 2)

- The Pearson correlation coefficient, r, is a measure of the linear association between two variables for which interval or ratio data are available.

- The Spearman rank-correlation coefficient, r_s , is a measure of association between two variables when only ordinal data are available.

- Values of r_s can range from –1 to +1, where
  - values near 1 indicate a strong positive association between the rankings, and
  - values near – 1 indicate a strong negative association between the rankings

Rank Correlation (2 of 2)

Spearman Rank-Correlation Coefficient, r_s

$$r_s = 1 - \frac{6\sum {d_i}^2}{n(n^2 - 1)}$$

where:

n = number of observations being ranked

x_i = rank of observation i with respect to the first variable

y_i = rank of observation i with respect to the second variable

d_i = $x_i - y_i$

Test for Significant Rank Correlation

We may want to use sample results to make an inference about the population rank correlation p_s.

- To do so, we must test the hypotheses:

H_0: $p_s = 0$ (No rank correlation exists)

H_a: $p_s \neq 0$ (Rank correlation exists)

Rank Correlation (1 of 7)

Sampling Distribution of r_s when p_s = 0

- Mean

$$\mu_{r_s} = 0$$

- Standard Deviation

$$\sigma_{r_s} = \sqrt{\frac{1}{n-1}}$$

- Distribution Form

Approximately normal, provided $n \geq 10$

Rank Correlation (2 of 7)

Example: Crennor Investors

Crennor Investors provides a portfolio management service for its clients. Two of Crennor's analysts ranked ten investments as shown on the next slide. Use rank correlation, with α = 0.10, to comment on the agreement of the two analysts' rankings.

Rank Correlation (3 of 7)

Example: Crennor Investors

- Analysts’ Rankings are shown in the table.
- Hypotheses

$$H_0: p_s = 0 \qquad \text{(No rank correlation exists)}$$
$$H_a: p_s \neq 0 \qquad \text{(Rank correlation exists)}$$

Investment	A	B	C	D	E	F	G	H	I	J
Analyst 1	1	4	9	8	6	3	5	7	2	10
Analyst 2	1	5	6	2	9	7	3	10	4	8

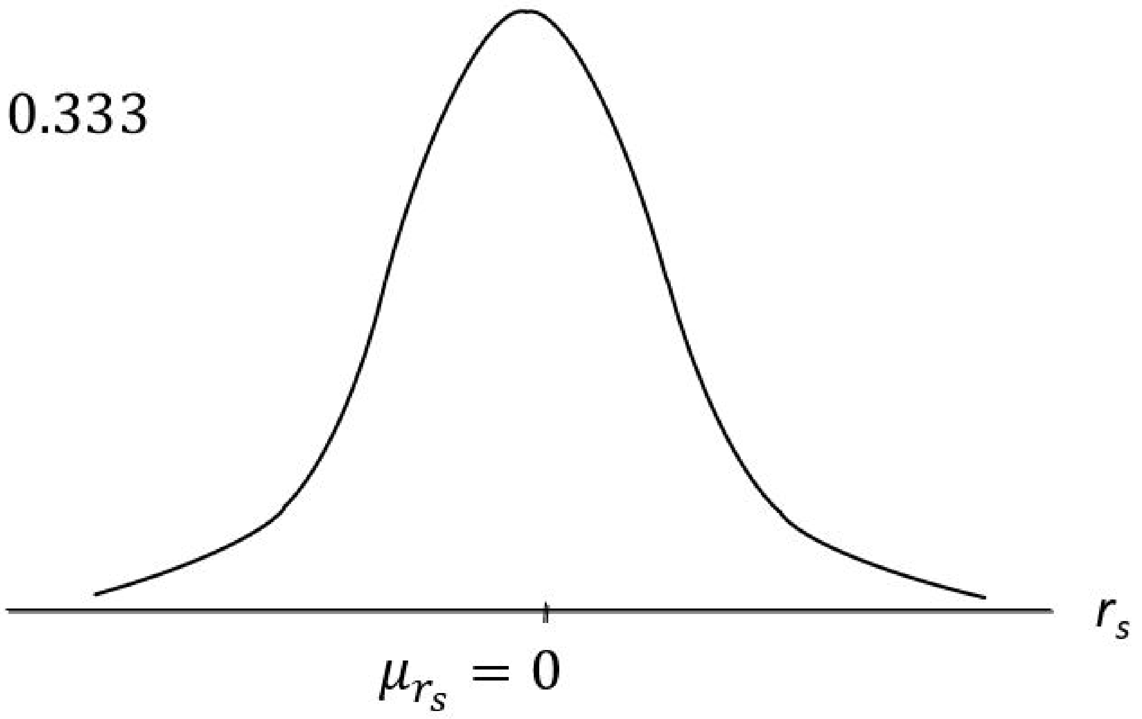
Rank Correlation (4 of 7)

Investment	Analyst #1 Ranking	Analyst #2 Ranking	Differ.	$(\text{Differ.})^2$
A	1	1	0	0
B	4	5	-1	1
C	9	6	3	9
D	8	2	6	36
E	6	9	-3	9
F	3	7	-4	16
G	5	3	2	4
H	7	10	-3	9
I	2	4	-2	4
J	10	8	2	4
			Sum =	92

Rank Correlation (5 of 7)

Sampling Distribution of r_s Assuming No Rank Correlation

$$\sigma_{r_s} = \sqrt{\frac{1}{10-1}} = 0.333$$



Rank Correlation (6 of 7)

- Rejection Rule

With 0.10 level of significance Reject H_0 if p-value $\leq$ 0.10

- Test Statistic

$$r_s = 1 - \frac{6\sum {d_i}^2}{n(n^2-1)} = 1 - \frac{6(92)}{10(100-1)} = 0.4424$$

$$z = \frac{r_s - \mu_r}{\sigma_r} = \frac{(0.4424 - 0)}{0.3333} = 1.33$$

- p-value

p-value = 2(1 – 0.9082) = 0.1836

Rank Correlation (7 of 7)

Conclusion

Do no reject H_0. The p-value > α. There is not a significant rank correlation. The two analysts are not showing agreement in their ranking of the risk associated with the different investments.