Understanding Patterns of Mortality Homogeneity and Heterogeneity across Countries and their Role in Modelling Mortality Dynamics and Hedging Longevity Risk

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Abstract

Understanding patterns of mortality homogeneity and heterogeneity across countries can assist in modelling mortality dynamics and in hedging longevity risk. This study proposes a methodology, based on the graduation method, to detect differences in mortality rates across different populations. Using an index $\hat{h}^2$ based on the Partial Standard Mortality Ratio (Lee, 2003), we measure mortality homogeneity and heterogeneity, then conduct an empirical study across countries with emerging and developed markets. The results of model fitting show that it is inappropriate to use a coherent mortality model for the mortality-heterogeneous populations. In an application, we demonstrate that a reinsurer can utilize information concerning mortality homogeneity/heterogeneity for pooling risk in its books of life insurance and annuity businesses and increase overall hedge effectiveness. The coherent mortality model can help reduce the volatility of the reinsurer's profit and help the reinsurer diversify its longevity risk.

Keywords: Longevity Risk; Hedge Effectiveness; Coherent Mortality Model; Standard Mortality Ratio

1. Introduction

Prolonged life expectancy is a common phenomenon of the twenty-first century. Modeling mortality rates thus is important for allowing actuaries to deal with longevity risk, and proper mortality models are needed to allow insurance companies to manage the longevity risk. As an option to transfer longevity risk, pooling insurance business across countries is promising, this in turn means that understanding the mortality pattern of the pooling insurance business can help increasing the effectiveness of longevity risk transfer. In particular, understanding patterns of mortality homogeneity or heterogeneity may help model mortality dynamics and manage longevity risk across countries. Similar mortality patterns arise among some population groups, which provides a means to handle modeling challenges, as noted in various studies.

For example, Lee and Nault (1993) investigate provincial mortality in Canada and suggest the use of the same general mortality rate level and age factors for each province in mortality forecasts. Lee (2000) tries to take advantage of similarities between men and women and impose coherence on the two sets of forecasts. Wilson (2001) argues that projections should assume global convergence in mortality, suggesting that it is inappropriate to prepare mortality forecasts for individual nations in isolation. Noting the convergence in life expectancy across 21 industrialized countries during the postwar period, White (2002) cites the increasing similarity in lifestyles in the wealthy world: The globalization of practices among rich countries affects mortality, leading to convergent mortality patterns. Li and Lee (2005) affirm that mortality patterns and trajectories in closely related populations are similar, and any differences are unlikely to increase in the long run. Therefore, without accounting for populations with similar mortality behaviors, Lee and Carter’s (1992) well-known model (LC model) may produce unreasonable results when applied to a single population (Tuljapurkar et al., 2000; Li and Lee, 2005). When Tuljapurkar et al. (2000) study mortality among the G7 countries separately, they find that the largest gap for projections of life expectancy, over a 50-year forecast horizon, increases from about 4 to 8 years—an implausible result. Li and Lee (2005) therefore caution against using the LC model for single countries and recommend applying it instead to groups of populations with similar socioeconomic conditions, to ensure stability in the parameter estimates and overcome divergence problems. That is, Li and Lee propose coherent mortality modeling under the LC model.

Several studies in turn suggest extending coherent mortality modeling. Yang et al. (2008) point out that coherent mortality modeling can solve the problem of insufficient mortality data when constructing mortality dynamics. Groups of populations also share some mortality factors that can ensure stability in the parameter estimates and mortality forecasting. They therefore propose a coherent LC model for the same gender across different countries that share common mortality time trends. Russolillo et al. (2011) extend the bilinear LC model and specify a new model based on a three-way structure, which incorporates another component to decompose the log mortality rates. To measure basis risk in longevity hedges, Li and Hardy (2011) propose four extensions to the LC model. Furthermore, Li (2013) examines the application of a Poisson common factor model to project mortality, jointly for women and men, which extends the structure of the LC model. Rather than a coherent LC framework, Hatzopoulos and Haberman (2013) propose a new common mortality modeling structure for analyzing mortality dynamics for a pool of countries, under the framework of generalized linear models. Villegas and Haberman (2014) take socio-economic mortality differentials into account to develop a relative model that allows for the simultaneous modeling of the mortality of a group of a subpopulation. Beyond coherent mortality modeling, another tactic for modeling the mortality rate for a particular segment of the population relies on time-series analysis with period effects.

The main reason the coherent mortality model works so well is the available sample size (Yang et al., 2008; Yue et al., 2015). The parameter estimates of the LC model grow more stable with the inclusion of data from populations with similar attributes. In practice though, it is not easy to judge whether a group of populations is homogeneous or have similar mortality attributes. Nor is it likely that all age groups have similar or identical mortality improvements. The influences of mortality discrepancy among populations and age groups on parameter estimates and mortality forecasts remain unknown. In terms of practical considerations, we also do not know if certain mortality indices might help construct a coherent mortality model.

This study attempts to analyze mortality homogeneity or mortality heterogeneity in a group of populations to determine the usefulness of coherent mortality modeling as a means to hedge longevity risk. We first apply a methodology based on a graduation method to detect discrepancies across populations, specifically, the Partial Standard Mortality Ratio (PSMR) which proposed by Lee (2003). To test for mortality homogeneity or mortality heterogeneity, we adapt an explanation ratio from Li and Lee (2005) to evaluate the goodness of fit, then conduct an empirical investigation of mortality homogeneity or mortality heterogeneity across countries. The fit of the coherent mortality model with countries that exhibit mortality homogeneity or mortality heterogeneity then can be compared. For the coherent mortality model, we consider both period and cohort effects and examine the coherent mortality framework under both the LC and Renshaw and Haberman (2006) models. In addition, we account for information about mortality homogeneity or mortality heterogeneity to consider the risk pooling strategy for a reinsurance company and accordingly examine hedge effectiveness.

The empirical study aims to measure mortality homogeneity or mortality heterogeneity across Asian countries and nearby developed countries, such as Australia and New Zealand. Although several nations in Asia have enjoyed rapid economic growth and increased living standards in recent years, many countries in this area still are considered emerging markets, despite having some developed market characteristics. Thus for example, analysts continue to categorize Taiwan and South Korea as emerging markets.[1] Longevity risk in these emerging countries is a critical issue, because people's life expectancy has improved significantly. Therefore, across Taiwan, South Korea, Hong Kong, Singapore, Japan, Australia, and New Zealand, we identify mortality patterns. Specifically, we find that for men, the mortality patterns in Australia and New Zealand are the most

[1]See "MSCI Emerging Markets Indexes," Retrieved 2015-02-02.

homogeneous, whereas the comparison of Australia and Taiwan reveals the most heterogeneity. For women, Australia and Hong Kong are the most homogeneous, and Singapore and New Zealand are the most heterogeneous. Then, when we fit the coherent models to the selection of the homogeneous mortality groups, the goodness of fit of the mortality model improves.

As an application, we investigate the effectiveness of a risk pooling strategy for reinsurance that addresses whether mortality is homogeneous or heterogeneous. Our results show that selecting a population group with homogeneous or heterogeneous mortality has different influences on the effectiveness of risk pooling. That is, a heterogeneous mortality pattern across pooled countries can diversify the risk associated with the same type of reinsurance policies. Reinsurers also can reduce their longevity risk by pooling both annuity and insurance policies, which constitutes a natural hedging strategy. In this case, a homogeneous mortality group can reduce risk more than a heterogeneous mortality group with a natural hedging reinsurance strategy.

This research accordingly makes several contributions to coherent mortality modeling and efforts to manage longevity risk. First, we provide a standard for measuring the characteristics of mortality, which improves the precision of mortality models across countries. This measure is based on the PSMR which detects discrepancies across populations, and we can identify mortality homogeneity or mortality heterogeneity.

Second, extending existing literature on coherent mortality modeling, we consider both period and cohort effects in the coherent framework with Lee and Carter’s (1992) and Renshaw and Haberman’s (2006) models. Our empirical study demonstrates that the selection of the countries with more similar mortality patterns can increase the goodness of fit of the coherent model. The coherent mortality model with both period and cohort effects also produces a better fitting result. Third, we study the hedging effectiveness of pooling risk across countries with mortality-homogeneous or mortality-heterogeneous groups. Understanding the characteristics of mortality can help reinsurers diversify their longevity risk across countries and increase their hedge effectiveness.

In Section 2, we present single and coherent models, as well as introducing the graduation method we use, PSMR. Section 3 reports the data, mortality dynamics, and empirical evaluation of why PSMR works well for selecting coherent group, using the explanation ratio. The application for reinsurance companies, using mortality homogeneity or mortality heterogeneity characteristics to inform risk pooling across countries and increase their hedge effectiveness, is the focus in Section 4. Finally, we offer a discussion and suggestions for coherent modeling in Section 5.

2. Methodology

2.1. Coherent mortality models

To examine mortality patterns in different populations for coherent mortality modeling, we consider two types of coherent frameworks, extended from Lee and Carter's (1992) and Renshaw and Haberman's (2006) models. The former capture the common trend of mortality across countries, depending on the period effect; the latter includes both period and cohort effects to model the common trend. For example, let $m_{ij}(x,t)$ denote the central death rate for a person aged x at time t for the ith gender in the jth country. Under the LC model, the mortality dynamic can be captured as

$$\text{LC:}\quad \ln(m_{ij}(x,t)) = \alpha_{ij}(x) + \beta_{ij}^{1}(x)\kappa_{ij}(t) + \varepsilon_{ij}(x,t)\,, \tag{1}$$

which captures the age–period effect for the ith gender in the jth population with an $\alpha_{ij}(x)$ coefficient, and the force of mortality for the ith gender in the jth population changes according to an overall mortality index $\kappa_{ij}(t)$, which is modulated by an age response $\beta_{ij}^{1}(x)$. The error term $\varepsilon_{ij}(x,t)$ denotes the deviation of the model from the observed log-central death rates and is assumed to be white noise, with a zero mean. In addition, the parameters $\beta_{ij}^{1}(x)$ and $\kappa_{ij}(t)$ are subject to $\sum_x \beta_{ij}(x) = 1$ and $\sum_t \kappa_{ij}(t) = 0$; these two conditions ensure model identification.

Li and Lee (2005) propose the first coherent mortality model under the LC framework by considering populations within a group that have the same age factor of $\beta_{ij}^1(x)$ and the same mortality time trend of $\kappa_{ij}(t)$. That is,

$$\text{LL:} \quad \ln(m_{ij}(x,t)) = \alpha_{ij}(x) + B^1(x)K(t) + \varepsilon_{ij}(x,t). \tag{2}$$

The change over time in mortality for different populations can be described by the term $B^1(x)K(t)$, where $B^1(x)$ and $K(t)$ are common factors that capture the age and period effects for each population in the group. In contrast, in the classical LC model in Equation (1), the term $\beta_{ij}^1(x)\kappa_{ij}(t)$ denotes the specific attribute for the population composed of the *i*th gender in the *j*th country.

In addition to the LL model, we take the cohort effect into account for coherent mortality modeling, extending from Renshaw and Haberman's (2006) model (or RH model). The original RH model is defined as

$$\text{RH:} \quad \ln(m_{ij}(x,t)) = \alpha_{ij}(x) + \beta_{ij}^1(x)\kappa_{ij}(t) + \beta_{ij}^2(x)\gamma_{ij}(c) + \varepsilon_{ij}(x,t), \tag{3}$$

where $(\alpha_{ij}(x), \beta_{ij}^1(x), \beta_{ij}^2(x))$ represent the age effect, $\kappa_{ij}(t)$ indicates the period effect, and $\gamma_{ij}(c)$ is the cohort effect of the *i*th gender in the *j*th country. The coherent mortality modeling under the RH framework (or RHC model) then becomes

$$\text{RHC:} \quad \ln(m_{ij}(x,t)) = \alpha_{ij}(x) + B^1(x)K(t) + B^2(x)\Gamma(c) + \varepsilon_{ij}(x,t), \tag{4}$$

where $(B^1(x), B^2(x))$, $K(t)$, and $\Gamma(c)$ represent the common factors with respect to age, period, and cohort, respectively, for a group of populations. With this model, each population has its own base mortality level but shares the same change rate, in terms of time and cohort.

However, it is not easy to predict the common factors in coherent mortality modeling, because of the assumption that the countries which are combined have the same common factors. Therefore, when the LL or RHC model is used, we face the problem of how to select the populations. Understanding the patterns of mortality homogeneity and heterogeneity might help to solve this problem. Accordingly, we attempt to detect mortality patterns for coherent mortality modeling. We then evaluate the goodness of fit of the resulting coherent mortality model and compare it against both the LC and RH models.

2.2 Detecting mortality homogeneity or mortality heterogeneity

To select the populations for our coherent mortality modeling, we use graduation methods to detect mortality homogeneity across populations. Specifically, we make use of an index $\hat{h}^2$ which is based on the Partial Standard Mortality Ratio (PSMR) proposed by Lee (2003). The PSMR is a modification of SMR. A SMR greater or less than 1 indicates an area that has a higher or lower overall mortality rate, respectively. The SMR is defined as

$$\mathrm{SMR} = \frac{\sum_x d_{ij}(x,t)}{\sum_x E_{ij}(x,t)} \tag{5}$$

where $E_{ij}(x,t)$ represents the expected death count if the study population assumes the standard rates, and $d_{ij}(x,t)$ is the observed death number in the study population group for a person aged x for the *i*th gender in the *j*th country at time t. To be more precise, we expect that countries may have the same mortality rate pattern, thus, we can define that $E_{ij}(x,t)$ is equal to $L_{ij}(x,t) \cdot \frac{d_{ij}^*(x,t)}{L_{ij}^*(x,t)}$ where $L_{ij}(x,t)$ is the study population number, $L_{ij}^*(x,t)$ is the standard population number, and $d_{ij}^*(x,t)$ is the observed death number in the standard population group for a person aged x for the *i*th gender in the *j*th country at time t.

Moreover, Lee (2003) defines $\hat{h}^2$ to measure heterogeneity in mortality rates between populations as follows:

$$\hat{h}^2 = \max(0, \frac{\sum_x[\left(d_{ij}(x,t) - E_{ij}(x,t)\cdot\mathrm{SMR}\right)^2 - \sum_x d_{ij}(x,t)]}{\mathrm{SMR}^2 \cdot \sum_x E_{ij}(x,t)}) \tag{6}$$

Therefore, the larger $\hat{h}^2$ is, the greater the difference in age-specific mortality rates (i.e., greater dissimilarity in the shapes of an age-specific mortality curve between the populations, or mortality heterogeneity). This quantity can be used as a measure to determine if two populations are homogeneous. We explore its effectiveness empirically in this study.

2.3 Model fit criterion

To evaluate the goodness of fit of the coherent mortality model, we apply the explanation ratio proposed by Li and Lee (2005). Let $R(ij)$ denote the explanation ratio for the ith gender in the jth country. Then, the explanation ratios for the preceding four models shown in Equations (1)-(4) can be calculated as:

$$R(ij)^{\mathrm{LC}} = 1 - \frac{\sum_t \sum_x [\ln\left(m_{ij}(x,t)\right) - \alpha_{ij}(x) - \beta_{ij}^1(x)\kappa_{ij}(t)]^2}{\sum_t \sum_x [\ln\left(m_{ij}(x,t)\right) - \alpha_{ij}(x)]^2},$$

$$R(ij)^{\mathrm{LL}} = 1 - \frac{\sum_t \sum_x [\ln\left(m_{ij}(x,t)\right) - \alpha_{ij}(x) - B^1(x)K(t)]^2}{\sum_t \sum_x [\ln\left(m_{ij}(x,t)\right) - \alpha_{ij}(x)]^2},$$

$$R(ij)^{\mathrm{RH}} = 1 - \frac{\sum_t \sum_x [\ln\left(m_{ij}(x,t)\right) - \alpha_{ij}(x) - \beta_{ij}^1(x)\kappa_{ij}(t) - \beta_{ij}^2(x)\gamma_{ij}(c)]^2}{\sum_t \sum_x [\ln\left(m_{ij}(x,t)\right) - \alpha_{ij}(x)]^2}, \text{ and}$$

$$R(ij)^{\mathrm{RHC}} = 1 - \frac{\sum_t \sum_x [\ln\left(m_{ij}(x,t)\right) - \alpha_{ij}(x) - B^1(x)K(t) - B^2(x)\Gamma(c)]^2}{\sum_t \sum_x [\ln\left(m_{ij}(x,t)\right) - \alpha_{ij}(x)]^2}.$$

If $R(ij)$ is too small, it implies that the model's factors may not be good enough to explain the mortality dynamics.

3. Empirical Study: Mortality Homogeneity or Heterogeneity and Model Fit

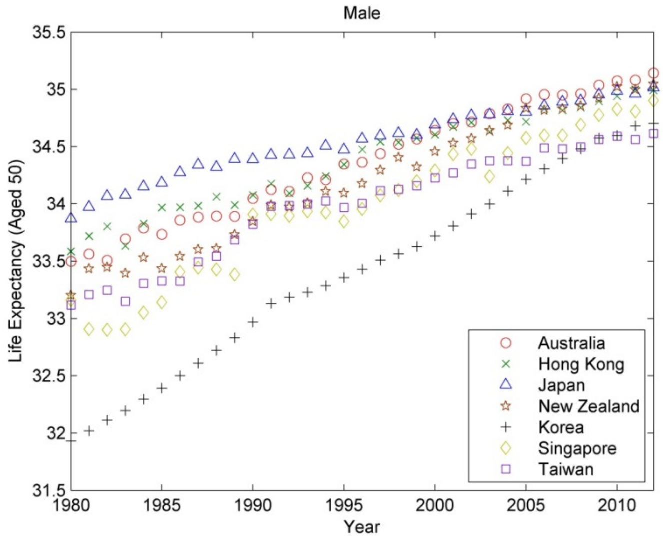
3.1 Data description

In the empirical study, we measure mortality homogeneity or mortality heterogeneity across the Asian countries and the nearby developed countries of Taiwan, South Korea, Hong Kong, Singapore, Japan, Australia, and New Zealand. We employ a data set gathered by the Insurance Risk and Finance Research Centre, at Nanyang Business School,[2] which includes 18 countries[3] in

[2] The data collection efforts focused on The Human Mortality Database (HMD), Department of Statistics, Ministry of Health Sample registration system (for India), World Bank, and other sources. It is freely available on the Centre's website (www.irfrc.com).

[3] Australia, Bangladesh, China, Hong Kong, India, Indonesia, Japan, Malaysia, Nepal, New Zealand, Pakistan,

the Asia-Pacific region. We focus on data pertaining to persons aged 50–79 years, for the data period from 1980 to 2012, which represent the longest spans of available year and age information in data set, and we assume that the coherent convergence is continued after 2012. The details of the data collection are available in Milidonis (2015). The mortality patterns of life expectancy in these countries, as presented in Figure 1, show that mortality rates for women improved faster than those for men. For example, a person of age 50 in 1980 in Australia was expected to live 34.8 years if female but 33.5 years if male. By 2012, life expectancy increased to 35.6 years for women and 35.1 years for men. Despite the differences, the extended lifespans were significant for both genders. Australian men and Japanese women have achieved the longest life expectancies in recent years.



Philippines, Republic of Korea, Singapore, Sri Lanka, Taiwan, Thailand, and Vietnam. The search was conducted at the end of 2012.

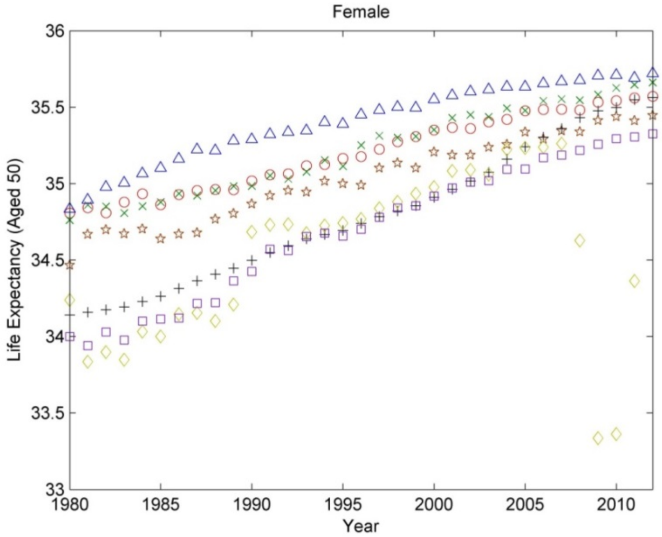


Figure 1. Mortality patterns of life expectancy in selected countries

(Top: Males; Bottom: Females)

3.2 Detecting Mortality homogeneity or heterogeneity

We examine mortality homogeneity or heterogeneity across countries by calculating the $\hat{h}^2$ ratios. The $\hat{h}^2$ ratios for different countries are plotted in Figure 2, and the reference population is Australia males. The corresponding average $\hat{h}^2$ ratios between two countries are shown in Tables 1 and 2. A large average $\hat{h}^2$ ratio implies the two countries are mortality heterogeneous and vice versa. For the male population, the mortality pattern between Australia and New Zealand reflects the most homogeneous country grouping, with the lowest average $\hat{h}^2$ ratio of 0.000674. The mortality rates of Australia and Taiwan instead turn out to be the most heterogeneous, with the highest average $\hat{h}^2$ ratio of 0.012420. Among the female population, Australia and Hong Kong are the most homogeneous country group, with the low average $\hat{h}^2$ ratio of 0.002002, whereas Singapore and New Zealand are the most heterogeneous, with the highest average $\hat{h}^2$ ratio of 0.048700.

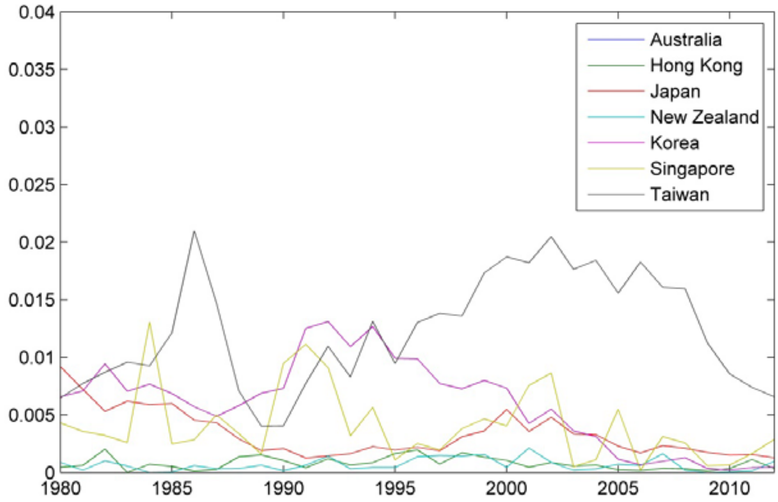


Figure 2 The $\hat{h}^2$ ratios in different years: males

Table 1 The average $\hat{h}^2$ ratios of the selected two countries: males

	Australia	Hong Kong	Japan	New Zealand	Korea	Singapore	Taiwan
Australia	0	—	—	—	—	—	—
Hong Kong	0.000799	0	—	—	—	—	—
Japan	0.003383	0.003263	0	—	—	—	—
New Zealand	**0.000674**	0.000675	0.003141	0	—	—	—
Korea	0.005738	0.003110	0.007170	0.004686	0	—	—
Singapore	0.004078	0.002746	0.004846	0.003226	0.005123	0	—
Taiwan	**0.012420**	0.010614	0.004834	0.010609	0.010126	0.010059	0

Table 2 The average $\hat{h}^2$ ratios for females

	Australia	Hong Kong	Japan	New Zealand	Korea	Singapore	Taiwan
Australia	0	—	—	—	—	—	—
Hong Kong	**0.002002**	0	—	—	—	—	—
Japan	0.003755	0.005844	0	—	—	—	—
New Zealand	0.003266	0.00826	0.009265	0	—	—	—
Korea	0.006092	0.002498	0.011599	0.014764	0	—	—
Singapore	0.043419	0.045105	0.044685	**0.048700**	0.045703	0	—
Taiwan	0.004018	0.005263	0.002255	0.008992	0.010307	0.040089	0

We next investigate whether this information about mortality homogeneity or heterogeneity across countries can benefit coherent mortality modeling.

3.3 Model fit

We evaluate the goodness of fit of the coherent model when it contains information about mortality homogeneity or mortality heterogeneity. In line with previous empirical results, we show that Australia and New Zealand as a mortality-homogenous group and Australia and Taiwan are a mortality-heterogeneous group for men. We examine the coherent mortality framework under both the LC model and the RHC model, using the explanation ratio ($R(ij)$) proposed by Li and Lee (2005) to examine their fit. For comparison purposes, we examine four models: LC, LL, RH, and RHC (see Section 2). The parameter estimates are in Appendix A.

The explanation ratios of these four models, as detailed in Table 3, offer some interesting results. The explanation ratios do not change much for the group of homogeneous countries under either single or coherent mortality modeling. They all exceed 97%. Therefore, the explanation ratios for homogeneous countries are not affected much by whether we model dynamics coherently or singly. In contrast, the explanation ratios for mortality-heterogeneous groups in coherent models are smaller than those of both mortality-homogeneous groups and single models. For example, comparing the single and coherent models for the mortality-heterogeneous groups, the explanation ratio for Australia is 0.998246 in the RH model but only 0.914093 in the RHC model. In Taiwan, we find another significant difference, with explanation ratios of 0.990588 and 0.890406 for the RH and RHC models, respectively. Similar results apply to the LC and LL models. The explanation ratio of the coherent model thus emerges as smaller when we combine heterogeneous mortality groups, implying that it is improper to use the coherent mortality model for these populations.

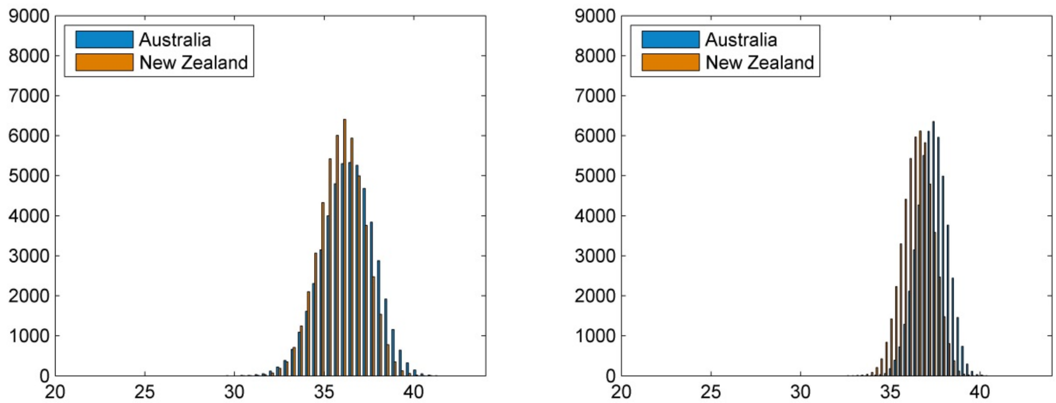
The effect of coherent modeling when combining a heterogeneous-mortality group or mortality-homogeneous can be depicted in Figure 2. For simplicity and without loss of generality, Figure 3 shows the distribution of the simulated life expectancy for men aged 50 years for example.[4] By comparing the RH and RHC models, we determine that the coherent model can help reduce volatility when mortality is homogeneous, but it has a completely contrary effect when mortality is heterogeneous. Specifically, the standard deviations of life expectancy in

[4] We ran the simulation 5,000 times to forecast life expectancy.

mortality-homogeneous groups are smaller than those of heterogeneous groups in the RHC model. Accordingly, the models become more stable when we include data from populations with similar attributes. Such effects are similar for the coherent mortality modeling under LC model.

Table 3 Explanation ratio for mortality homogeneity and heterogeneity groups (Males)

Model	Mortality homogeneity		Mortality heterogeneity		Ratio
	Country	$R(ij)$	Country	$R(ij)$	$R^*(ij)$[5]
LC	Australia	0.980332	Australia	0.980332	1.025791
	New Zealand	0.985947	Taiwan	0.936510	
LL	Australia	0.977144	Australia	0.892099	1.180661
	New Zealand	0.985560	Taiwan	0.770278	
RH	Australia	0.998246	Australia	0.998246	1.001716
	New Zealand	0.994001	Taiwan	0.990588	
RHC	Australia	0.993845	Australia	0.914093	1.099775
	New Zealand	0.990698	Taiwan	0.890406	



[5] $R^*(ij)$ is mortality homogeneity's $R(ij)$ divided by mortality heterogeneity's $R(ij)$. Therefore, if $R^*(ij)$ over 1 then the performance of mortality homogeneity is better than mortality heterogeneity, vice versa.

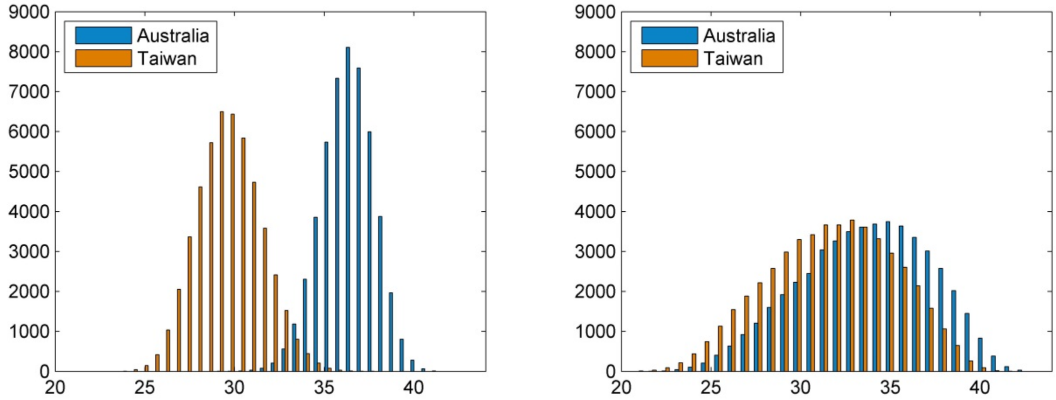


Figure 3. Distribution of life expectancy, men, aged 50 years (Left: RH mortality model; Right: RHC mortality model; Top: mortality-homogeneous group; Bottom: mortality heterogeneous group)

The fitting accuracy of a coherent mortality model may improve due to the increase of sample size. For a robustness check, we further discuss the coherence effect in relation to sample size in greater depth, to determine if a group with a large population can help 'stabilize' a group with a small population in terms of coherent mortality modeling. Among the mortality groups shown in Table 3, New Zealand and Taiwan have smaller populations compared to Australia. Therefore, we treat these two countries as the small population group, by adjusting the number of deaths and exposure to risks. These adjusted mortality distributions are identical to the unadjusted population's distributions, because the number of deaths and exposure to risks decrease on the same scale. For illustration, we divide the number of deaths and exposure to risks by 10 and 20, respectively.

The results for the explanation ratios according to the adjusted mortality distributions are in Table 4. A small population can take advantage of a large population when the mortalities in the two countries are homogeneous. The mortality estimation for the small population becomes more stable when we combine it with data from the large population, assuming the mortality rates in the small and large populations are homogeneous. However, the fit of the coherent mortality model for the small population becomes worse if they are heterogeneous. In addition, the fit diminishes with the population size in the heterogeneous groups. For example, the $R(ij)$ under the RHC model is

0.981055 in New Zealand and 0.175605 in Taiwan, according to the population adjusted by a scale of 10; the $R(ij)$ under the RHC model is 0.981025 in New Zealand and 0.092769 in Taiwan when the adjusted population is decreased by a scale 20. Coherent mortality modeling for the small populations therefore demands care, especially when mortality in the population group is heterogeneous. Our investigation affirms Yang et al.'s (2008) and Yue et al.'s (2015) claims that a small population achieves better fit in coherent (LL and RHC) rather than single (LC and RH) models. However, this effect does not apply to a group with mortality heterogeneity.

Table 4 Explanation ratio for mortality homogeneity and heterogeneity groups, according to the adjusted population (Male)

Model	Mortality homogeneity			Mortality heterogeneity			Ratio	
	Country	$R(ij)$ adjusted on the scale of 10	$R(ij)$ adjusted on the scale of 20	Country	$R(ij)$ adjusted on the scale of 10	$R(ij)$ adjusted on the scale of 20	$R^*(ij)$ adjusted on the scale of 10	$R^*(ij)$ adjusted on the scale of 20
LC	Australia	0.980332	0.980332	Australia	0.980332	0.980332	1.025857	1.025962
	New Zealand	0.985903	0.985904	Taiwan	0.936344	0.936149		
LL	Australia	0.980003	0.980180	Australia	0.978935	0.980703	1.638907	1.770882
	New Zealand	0.982691	0.982256	Taiwan	0.218628	0.127466		
RH	Australia	0.998246	0.998246	Australia	0.998246	0.998246	1.006122	1.005994
	New Zealand	0.993856	0.993600	Taiwan	0.981734	0.981732		
RHC	Australia	0.997877	0.998022	Australia	0.993675	0.996840	1.692436	1.816291
	New Zealand	0.981055	0.981025	Taiwan	0.175605	0.092769		

4. Application: Risk Pooling for Reinsurers Considering Mortality Homogeneity or Heterogeneity

4.1 Reinsurance strategy

Reinsurance is an important tool for insurers to manage their risks. It can help them reduce their insurance risks and the risks of insolvency, but to be effective, a reinsurer must handle its risk from reinsurance carefully. Risk pooling is critical to risk diversification. To deal with longevity risk, a reinsurer can pool insured risk from different countries. We thus investigate how effective risk pooling might help a reinsurance company diversify longevity risk. In other words, can reinsurers use information about mortality homogeneity or heterogeneity to diversify their reinsurance risk?

A reinsurer can pool its risk across product lines and populations. Imagine there are two reinsurers that want to investigate a product mix reinsurance strategy by pooling life insurance and annuity policies across different countries. One reinsurer uses two mortality-homogeneous countries, and the other uses two mortality-heterogeneous countries. To measure the effect of pooling risk, we first define the profit function for the reinsurer. The profit of life insurance contracts or annuities can be viewed as the difference between the actuarial present value, calculated according to the pricing mortality table, and actual mortality experience. The profit function (π_{ij}^{k}) for each type of insurance policy for the *i*th gender in the *j*th country is

$$\pi_{ij}^{k}(x) = \mathrm{APV}^{k}\left(x, q_{ij}(x,t)\right) - \mathrm{APV}^{k}\left(x, q_{ij}^{*}(x,t)\right), \qquad (7)$$

where k denotes the policy type (i.e., life insurance policy or life annuity policy), x is the age at which the profit function is calculated, APV^{k} represents the corresponding actuarial present value of the insurance policy, $q_{ij}(x,t)$ is the pricing mortality rate following the period life table in year t for the *i*th gender in the *j*th country, and $q_{ij}^{*}(x,t)$ is the actual mortality rate in year t for the *i*th gender in the *j*th country, captured by the dynamic stochastic mortality model investigated herein. For instance, for a man aged 50 in 2012, the mortality rates which we use will be $q_{ij}(50{,}2012)$, $q_{ij}(51{,}2012)$, …, $q_{ij}(100{,}2012)$ when we calculate the profit using the period life table; the mortality rates which we use will be $q_{ij}^{*}(50{,}2012)$, $q_{ij}^{*}(51{,}2013)$, …, $q_{ij}^{*}(100{,}2062)$ when we calculate the profit using the dynamic stochastic mortality model. Due to longevity risk, we expect a positive profit function for the life insurance policy but a negative profit function for the life annuity policy.

When pooling risk across countries, the profit function for the reinsurer represents the weighted profit for each profit in the product line for different countries, calculated as

$$H\left(\pi_{ij}^{k}\right) = \sum\nolimits_{\forall ijk} w_{ijk}\pi_{ij}^{k}(x), \qquad (8)$$

where $H\left(\pi_{ij}^{k}\right)$ is the profit function after hedging that uses the kth policy type for the *i*th gender in the *j*th country, w_{ijk} is the proportion of the kth policy type and *i*th gender in the *j*th country, and $\sum_{\forall ijk} w_{ijk} = 1$.

In the following numerical examples, we consider two types of policies in the reinsurance pool and two countries according to whether they form a mortality-homogeneous group or a mortality-heterogeneous group. Assume the annuity policy has an annual payment of $1 and the life insurance policy offers a benefit payment of $100. For comparison, we present the results under the RH and RHC model separately in the following analysis.[6]

4.2 The effect of risk pooling for annuity business across mortality-homogeneous countries vs. mortality-heterogeneous countries

In this section, we consider the reinsurers have annuity business across countries. The risk pooling effect across the mortality-homogeneous group and mortality-heterogeneous group are examined. For such purpose, consider two reinsurers A and B. Reinsurer A holds the risk from countries that exhibit mortality homogeneity, such as annuity business from Australia and New Zealand. Reinsurer B pools the risk from countries with mortality heterogeneity, such as the annuity business from Australia and Taiwan. The two reinsurers calculate their profit for the annuity policy using the coherent mortality model according to the mortality-homogeneous and mortality-heterogeneous groups. The distribution of the simulated profit and the corresponding statistics of the annuity policy in each country are shown in Figures 4, 5 and Table 5[7]. According to the simulated results, the mean, VaR(90), and CTE(90) are negative. It is clear that reinsurers face longevity risk when they reinsure the annuity business, whether this belongs to homogeneous or heterogeneous groups. In addition, the standard deviations of the simulated profit are smaller with

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[6] For each model, we run 5,000 simulations to forecast the future mortality rate. See Appendix B for the mortality forecast.

[7] We also calculate profit functions for men aged 60 and 70 years; the results are similar to those for 50 years. In the following numerical analysis, we only demonstrate for men aged 50 years.

the coherent mortality model for Reinsurer A, such as 0.724 and 0.399 in the RH and RHC, respectively, in Australia, and 0.627 and 0.413 in New Zealand. However, for Reinsurer B, the results are the opposite. The standard deviations in RHC are larger than in RH. Likewise, the fit and forecasting advantages of the coherent model emerge only in the mortality-homogeneous condition, but they suffer with the mortality-heterogeneous combinations. Therefore, information about mortality homogeneity or heterogeneity is important when combining different populations for the reinsurers.

In addition, the effect of pooling risk across countries can be seen in Figure 6. We assume the reinsurer pools 50% of the annuity policy in each country. The diversification of longevity risk with the annuity business can be reduced a little bit by pooling the risk with mortality-heterogeneous groups. For example, the VaR(90) equals -3.747 and -3.043 in the homogeneous and heterogeneous groups, respectively. Therefore, there are bigger losses from an annuities business in mortality-homogeneous groups than in mortality-heterogeneous groups.

Table 5 Simulated Profits of annuity business based on mortality-homogeneous group vs. mortality-heterogeneous group

Reinsurer	Country	Mortality Model	Mean	S.D.	Skewness	Kurtosis	VaR(90)	CTE(90)
A	Annuity	RH	-3.039	0.724	0.197	3.046	-3.951	-3.041
	(Australia)	RHC	-3.440	0.399	0.149	2.996	-3.946	-3.437
	Annuity	RH	-3.176	0.627	0.139	3.020	-3.970	-3.175
	(New Zealand)	RHC	-3.488	0.413	0.113	2.974	-4.015	-3.491
B	Annuity	RH	-3.039	0.724	0.197	3.046	-3.951	-3.041
	(Australia)	RHC	-1.872	1.719	0.298	2.577	-4.053	-1.853
	Annuity	RH	-2.048	0.863	-0.113	2.878	-3.171	-2.065
	(Taiwan)	RHC	-1.935	1.710	0.184	2.490	-4.146	-1.888

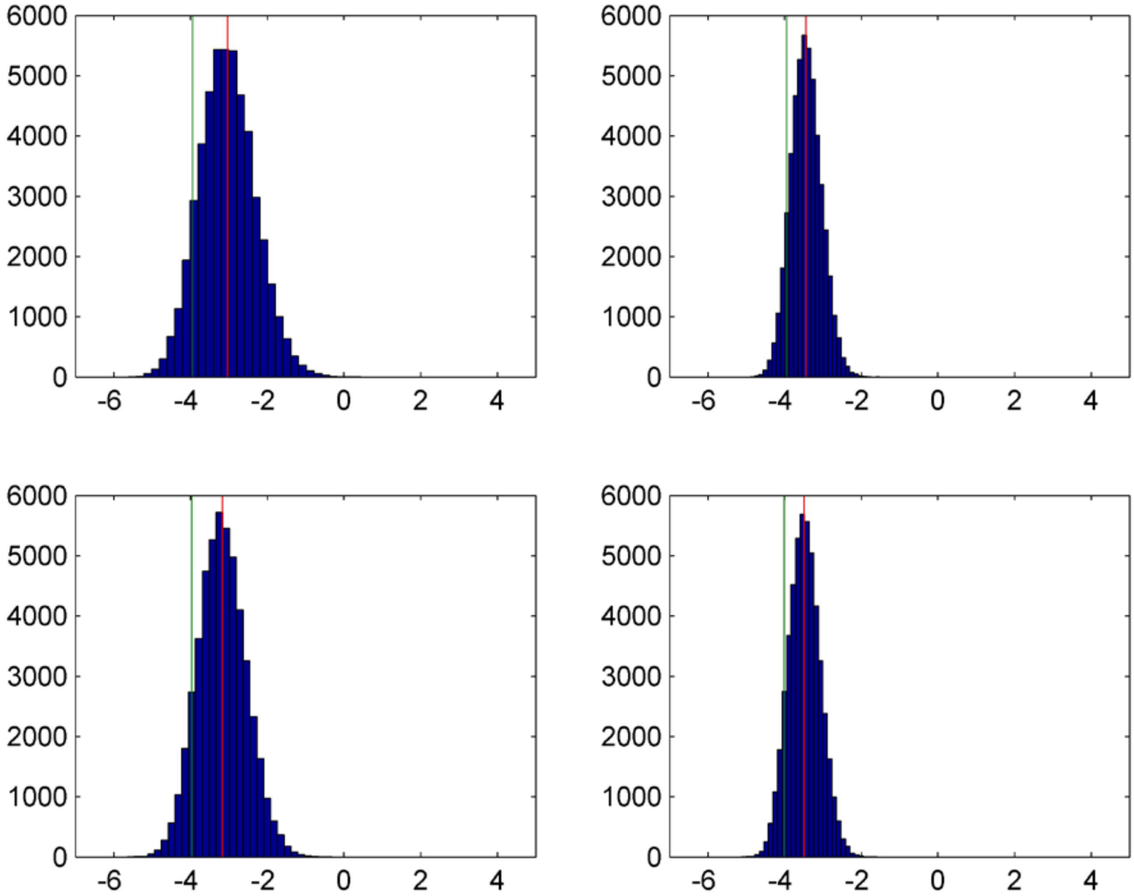
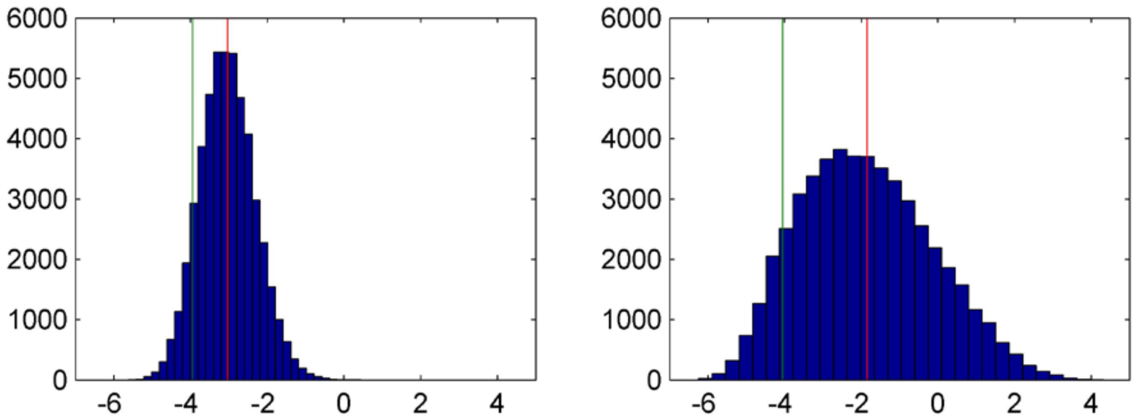


Figure 4. Simulated profit of annuities, men, aged 50 years in mortality-homogeneous groups (Left: RH mortality model; Right: RHC mortality model; Top: Australia; Bottom: New Zealand; Red line: VaR(90); Green line: CTE(90))



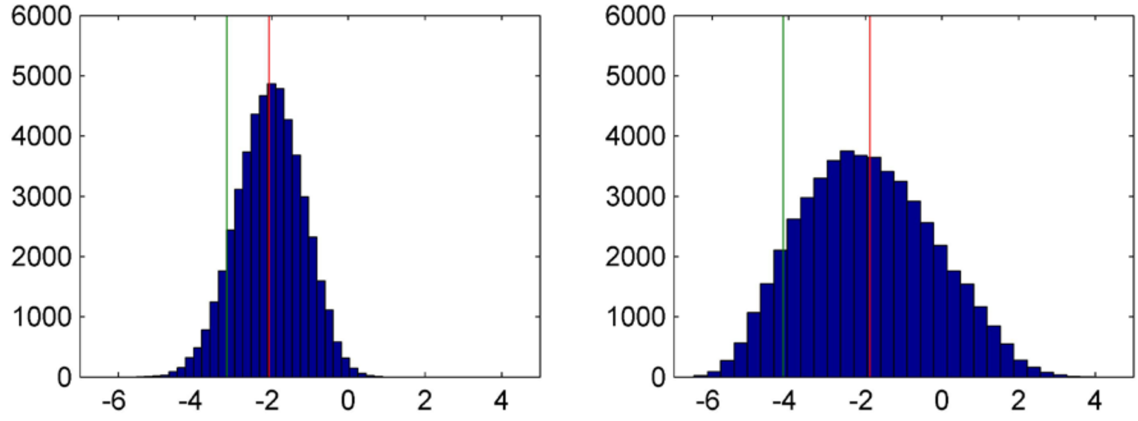


Figure 5. Simulated profit of annuities, men, aged 50 years in mortality- heterogeneous groups (Left: RH mortality model; Right: RHC mortality model; Top: Australia; Bottom: Taiwan; Red line: VaR(90); Green line: CTE(90))

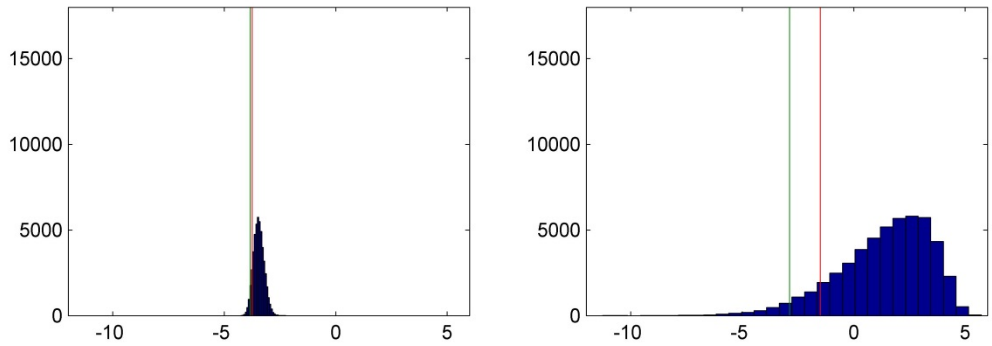


Figure 6 Hedging distribution, VaR(90), and CTE(90) in the mortality-homogeneous and mortality-heterogeneous groups (Left: mortality- homogeneous groups; Right: mortality-heterogeneous groups. Red line: VaR(90); Green line: CTE(90))

4.3 The effect of risk pooling for life insurance and annuity business across mortality-homogeneous countries vs. mortality-heterogeneous countries

In this section, we demonstrate the risk pooling effect when the reinsurers pool both insurance and the annuity business across countries. Since the impact of longevity risk on insurance and annuity policy are opposite. Such risk pooling works like the natural hedge strategy. The existing literature has found that a natural hedging strategy can diversify longevity risk. We intend to find that mortality homogeneity or heterogeneity may produce different levels of hedging efficiency.

Again we compare two reinsurers C and D. Reinsurer C pools risk from the countries with mortality homogeneity, such as by pooling the annuity and life insurances businesses from Australia and New Zealand separately. Reinsurer D instead considers countries with mortality heterogeneity and thus pools the annuity business from Australia and the life insurance business from Taiwan. The two reinsurers calculate their profit for the life insurance and annuity policy using the coherent mortality model according to the mortality-homogeneous and mortality-heterogeneous groups. The corresponding statistics of the simulated profit for the life insurance and annuity policy in each country are shown in Table 6. We also compare the hedge effectiveness of risk pooling across different product lines and countries in Figures 7, 8, 9. (The numerical values are in Appendix C.)

Table 6 offers several findings. First, according to the simulated profits, the mean, VaR(90), and CTE(90) are negative for annuity products, but they are positive in life insurance. Therefore, the reinsurer suffers longevity risk for its annuity business. Second, the standard deviations of the profits are smaller in the coherent mortality model of Reinsurer C. For example, they equal 0.724 and 0.399 for Australia in the RH and RHC models, respectively, but for New Zealand, the standard deviations for the same models are 0.414 and 0.317. In direct contrast, Reinsurer D presents results in which the standard deviations in the RHC are greater than those in the RH model. The fit and forecasting advantages of the coherent model also arise only in the mortality-homogeneous condition, whereas the results decline in quality when we combine mortality-heterogeneous countries. Therefore, if mortality is heterogeneous and we use a coherent model, it results in more profit volatility and demands larger reserves to deal with the greater risk.

Figures 7, 8, and 9 show that hedging strategies are affected by the mortality characteristics of different populations. We examine three different weights in these two countries in the hedging strategies: Hedging strategy I is 50% and 50%; hedging strategy II is 20% and 80%; and hedging strategy III is 80% and 20%.[8] Clearly, the hedging effectiveness of diversifying longevity risk through the annuity business can be increased by pooling risk with life insurance. Thus, hedging strategy II gives the best hedging effectiveness. For example, based on the RHC model, the mean profit is positive; 2.156 and 1.356 for the reinsurer C and D separately. VaR(90) increased from -3.946 to 1.734 for Reinsurer C and -4.053 to -1.496 for Reinsurer D. The coherent model also can take advantage of homogeneous mortality. For example, for Reinsurer C, the standard deviations of hedging strategy II are 0.475 and 0.324 in the RH and RHC models, respectively. The results for Reinsurer D again are completely different, which are 1.036 and 2.069 in the RH and RHC models. Therefore, the hedging strategy is affected by mortality homogeneity versus heterogeneity. In addition, the CTE(90) is consistently larger for Reinsurer C than for Reinsurer D, across all hedging strategies. In turn, Reinsurer D faces greater losses than Reinsurer C; the hedging efficiency of Reinsurer C is better than that of Reinsurer D, especially with regard to hedging strategy II. Reinsurer C even can reduce VaR(90) and CTE(90) until they become positive, whereas Reinsurer D still faces risk, because its VaR(90) and CTE(90) are always negative. Therefore, mortalities from different countries can help a reinsurance company diversify risk, but the effect is strongly limited when the pooled countries exhibit heterogeneous mortality.

Table 6 Simulated profits of annuity and life insurance business based on mortality-homogeneous group vs. mortality-heterogeneous group

	Country	Model	Mean	S.D.	Skewness	Kurtosis	VaR(90)	CTE(90)
C	Annuity	RH	-3.039	0.724	0.197	3.046	-3.951	-3.041
	(Australia)	RHC	-3.440	0.399	0.149	2.996	-3.946	-3.437
	Life Insurance	RH	3.203	0.414	-0.296	3.101	—	—
	(New Zealand)	RHC	3.555	0.317	-0.263	3.073	—	—
D	Annuity	RH	-3.039	0.724	0.197	3.046	-3.951	-3.041
	(Australia)	RHC	-1.872	1.719	0.298	2.577	-4.053	-1.853
	Life Insurance	RH	2.094	1.281	-0.198	2.971	—	—
	(Taiwan)	RHC	2.163	2.378	-0.982	4.068	—	—

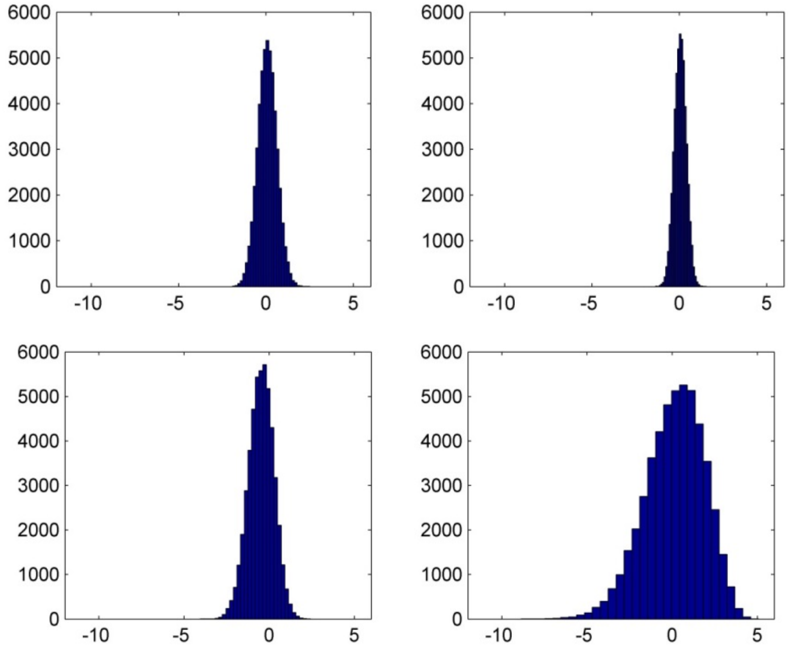


Figure 7 Pooling life insurance and annuity products for hedging strategy I (Upper left: Reinsurer C, RH; Upper right: Reinsurer C, RHC; Lower left: Reinsurer D, RH; Lower right: Reinsurer D, RHC)

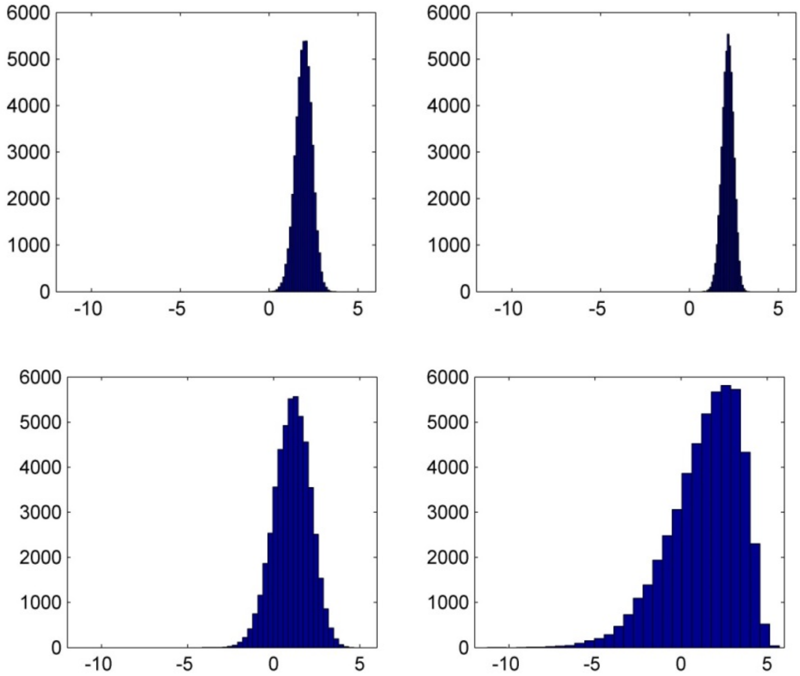


Figure 8 Pooling life insurance and annuity products for hedging strategy II (Upper left: Reinsurer C, RH; Upper right: Reinsurer C, RHC; Lower left: Reinsurer D, RH; Lower right: Reinsurer D, RHC)

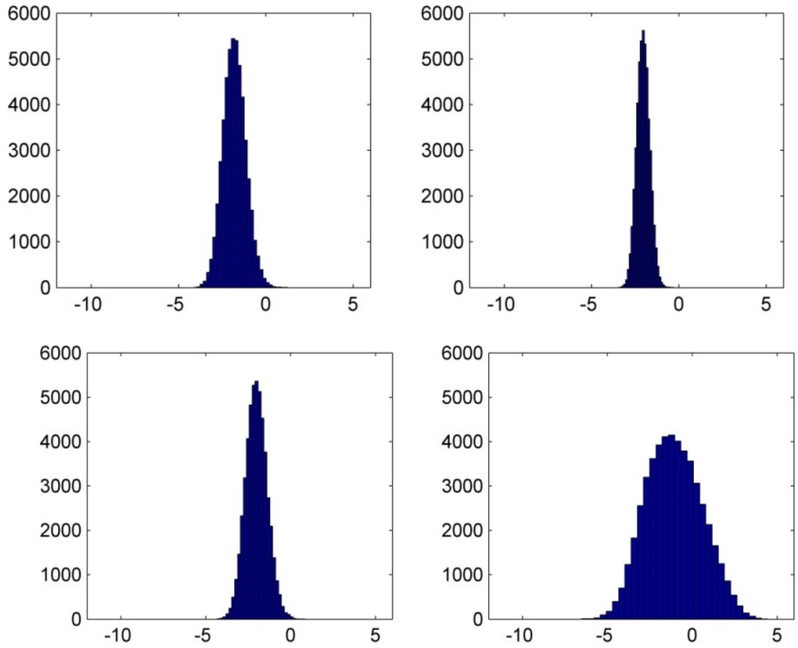


Figure 9 Pooling life insurance and annuity products for hedging strategy III (Upper left: Reinsurer C, RH; Upper right: Reinsurer C, RHC; Lower left: Reinsurer D, RH; Lower right: Reinsurer D, RHC)

5. Conclusions and Discussion

Mortality models are an important tool for dealing with longevity risk, but estimates of model parameters can be influenced by small sample sizes. Combining populations with similar mortality attributes is a feasible way of 'stabilizing' parameter estimates, but in practice, it can be difficult to judge whether a group of populations is homogeneous in terms of having similar mortality profiles. Not all age groups have similar mortality improvements, and it is difficult to predict the impact of discrepancies within a group of populations on the parameter estimates. In this study, we have explored mortality homogeneity and heterogeneity within a group of populations. We propose an approach to detect differences across populations, based on the Partial Standard Mortality Ratio (PSMR), one type of graduation method. To confirm the fit related to mortality homogeneity or mortality heterogeneity, we examine the model fit by calculating the explanation ratio (Li and Lee, 2005).

For our empirical investigation of mortality homogeneity or mortality heterogeneity between countries, we chose geographically proximate countries with similar economic conditions, to support the use of coherent modeling of mortality rates. Although several nations in Asia have enjoyed rapid economic growth and increased living standards in recent years, many countries in this area still are considered emerging markets, despite having some developed market characteristics. We used data from Singapore, Taiwan, Hong Kong, Korea, Japan, Australia, and New Zealand. In the case of male data, the mortality patterns in Australia and New Zealand are the most homogeneous, whereas the mortality patterns in Australia and Taiwan are the most heterogeneous. In turn, we show that explanation ratios would be smaller if we were to fit the coherent models with heterogeneous mortality groups. Moreover, we detect a big difference in the estimation between unadjusted and adjusted populations among the mortality-heterogeneous groups; the fit is affected by the population, but only for mortality-heterogeneous groups.

We also investigate the effectiveness of risk pooling life and annuity policies by reinsurance companies depending on whether policy holders are classified as belonging to mortality-homogeneous or mortality-heterogeneous populations. We analyze a stylized reinsurance company's profit function, solvency, longevity risk, and hedging strategies. We find a number of significant results. First, if mortality is heterogeneous but the reinsurance company uses a coherent model, it will experience greater volatility in its hedging profit and will need larger reserves to account for the greater risk. Therefore, the homogeneity or heterogeneity of mortality is critical and influences the efficacy of the reinsurer's capital. Second, the mean, VaR(90), and CTE(90) are negative for annuity policies, regardless of countries, ages, and models due to the risk of increasing life expectancy. Third, the outcome differs depending both on whether there is mortality homogeneity or heterogeneity and on whether reinsurance companies hedge the longevity risk in annuity policies with life insurance or with annuity policies from different countries. We find that using annuity policies to hedge longevity risk is less effective than using life insurance policies: in other words, life insurance policies are a much more effective 'natural hedge' for annuity policies than other types of annuity policies. We also find that hedging longevity risk in the annuity business is increased when hedging using annuities if the annuities come from mortality-heterogeneous groups. This is because the hedging instrument is the same as the underlying (i.e., both are types of annuities) and hence there are greater diversification benefits if the annuities come from mortality-heterogeneous groups. By contrast, we find that hedging longevity risk in the annuity business is increased when hedging using life insurance policies if the annuities come from mortality-homogeneous groups. This is because the correlation between the hedging instrument and the underlying will be higher and hence the hedge will be more effective. In other words: mortality-heterogeneous groups can diversify more longevity risk than mortality-homogeneous groups when the hedging instrument is the same as the underlying; mortality-homogeneous groups can diversify more longevity risk than mortality-heterogeneous groups when the hedging instrument is a 'natural hedge' for the underlying. Therefore, reinsurance companies can make use of information on the mortality-homogeneity and mortality-heterogeneity of the policies that they reinsure in the design of their hedging and capital utilization strategies.

This study thus serves as a first step toward a better understanding of the effect of mortality homogeneity and heterogeneity. While we provide a very simple case study for analyzing a reinsurance company's hedging effectiveness under different risk pooling strategies, we believe the framework can be used to design more complex strategies for hedging longevity risk in a reinsurer's book of business.

Appendix A: Parameter Estimation Procedures

Brillinger (1986) hypothesized that the number of deaths follows a Poisson distribution, $d_{ij}(x,t) \sim Poission(\lambda_{ij}(x,t))$, and the expected number of deaths in different populations is

$$\lambda_{ij}(x,t) = m_{ij}(x,t)L_{ij}(x,t).$$

The log-likelihood function under the LC and RH models for a single population can be expressed as:

$$L(\theta) = \sum d_{ij}(x,t)\ln(\lambda_{ij}(x,t)) - \lambda_{ij}(x,t) - \ln(d_{ij}(x,t)!). \text{ (A1)}$$

By solving Equation (A1), we find the parameter estimates $(\hat{\alpha}_{ij}(x), \hat{\beta}^1_{ij}(x), \hat{\kappa}_{ij}(t))$ in LC and $(\hat{\alpha}_{ij}(x), \hat{\beta}^1_{ij}(x), \hat{\kappa}_{ij}(t), \hat{\beta}^2_{ij}(x), \hat{\gamma}_{ij}(c))$ in RH, which can serve to project future mortality rates for each population.

Extending to the coherent models, we calculate the average death numbers for a group within a population, instead of single population, to find the coherent parameters. Let $d^c_{ij}(x,t)$ denote the average number of deaths at age *x* in year *t* for the *i*th gender in the *j*th country. The log-likelihood function under LL and RHC for a group of populations can be rewritten as

$$L(\theta^c) = \sum d^c_{ij}(x,t)\ln(\lambda_{ij}(x,t)) - \lambda_{ij}(x,t) - \ln(d^c_{ij}(x,t)!). \text{ (A2)}$$

For a group with N populations, the average number of deaths across populations is calculated as $d^c_{ij}(x,t) = \frac{1}{N}\sum_N d_{ij}(x,t)$, and the parameters are estimated as $(\hat{\alpha}_{ij}(x), \hat{B}^1(x), \hat{K}(t))$ in LL and $(\hat{\alpha}_{ij}(x), \hat{B}^1(x), \hat{K}(t), \hat{B}^2(x), \hat{\Gamma}(c))$ in RHC. The parameter estimation of the log-likelihood functions in Equations (A1) and (A2) can be solved recursively by the Newton method.

The parameter estimates of LL to RHC for two population groups of men in Australia and New Zealand (mortality homogeneity) and Australia and Taiwan (mortality heterogeneity) are plotted in Figures A1–A6. As expected, the mortality time and cohort trends in these four models decrease over time, indicating mortality rate improvements.

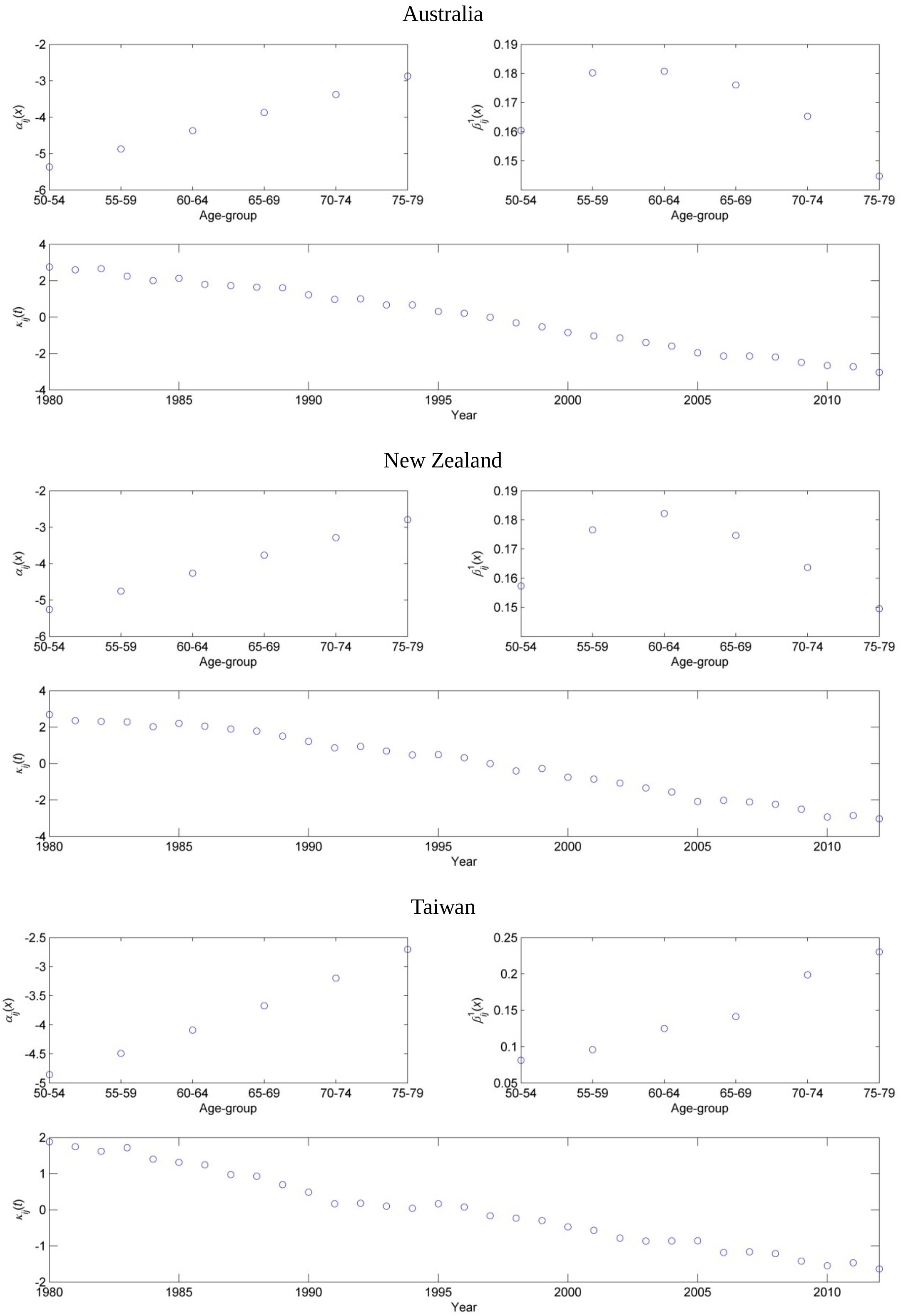
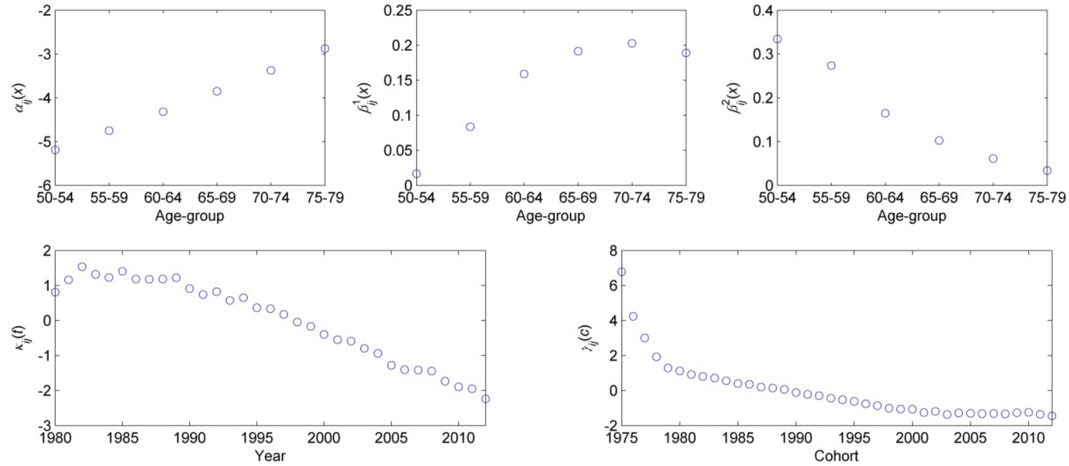
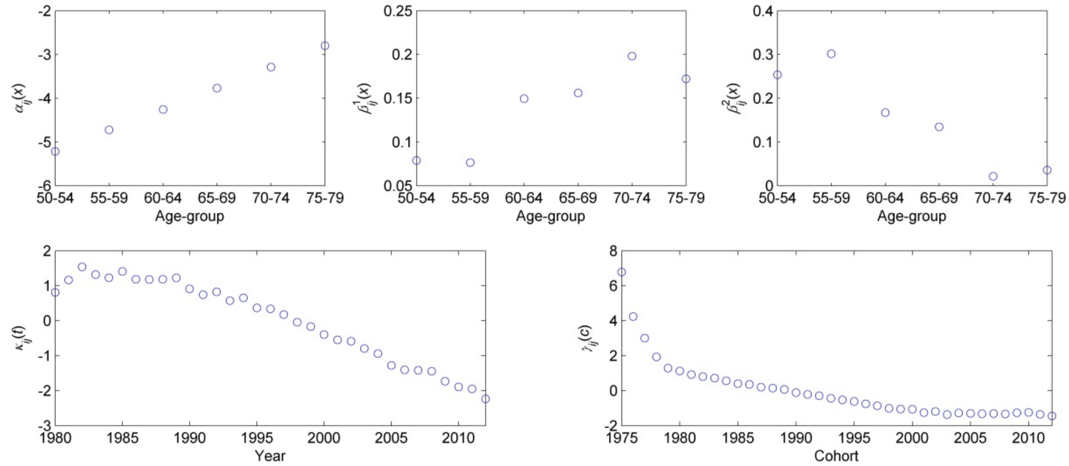


Figure A1 Parameter estimates, LC

Australia



New Zealand



Taiwan

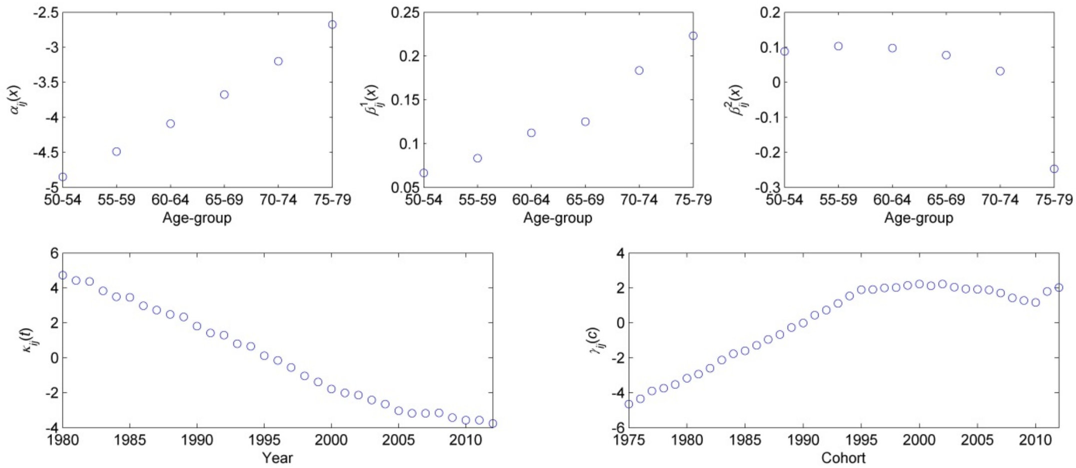
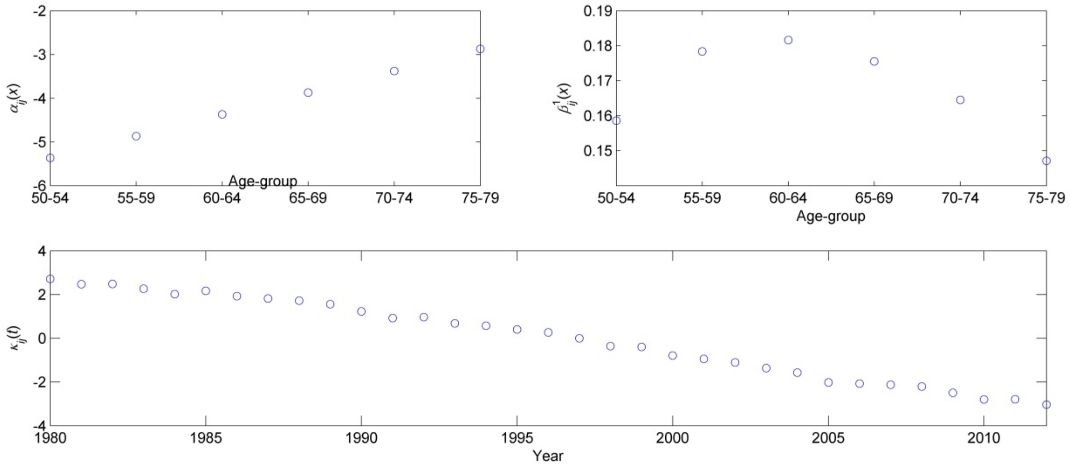


Figure A2 Parameter estimates, RH

Australia



New Zealand

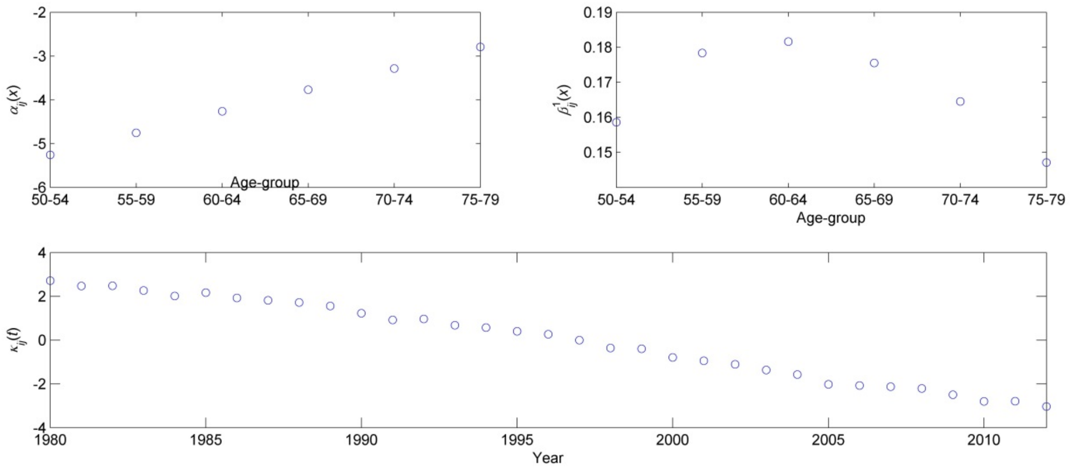
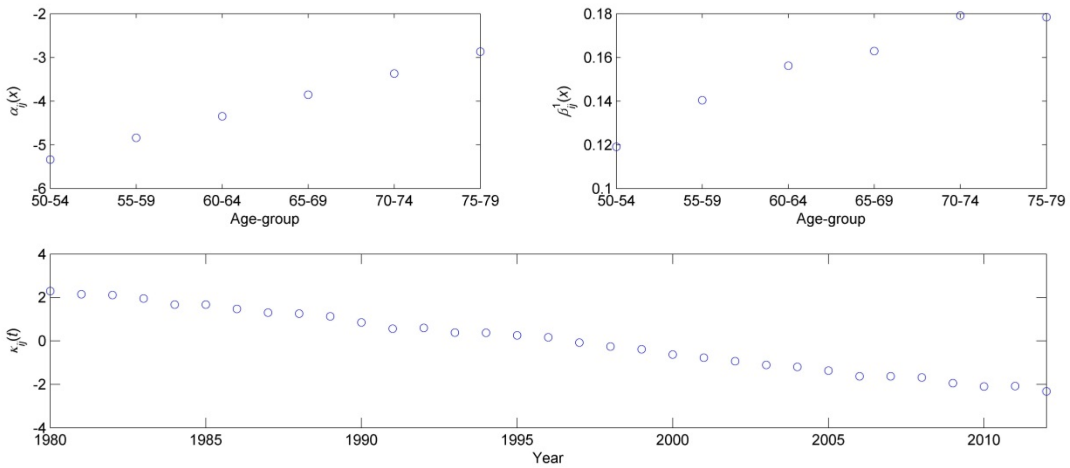


Figure A3 Parameter estimates in mortality homogeneity, LL

Australia



Taiwan

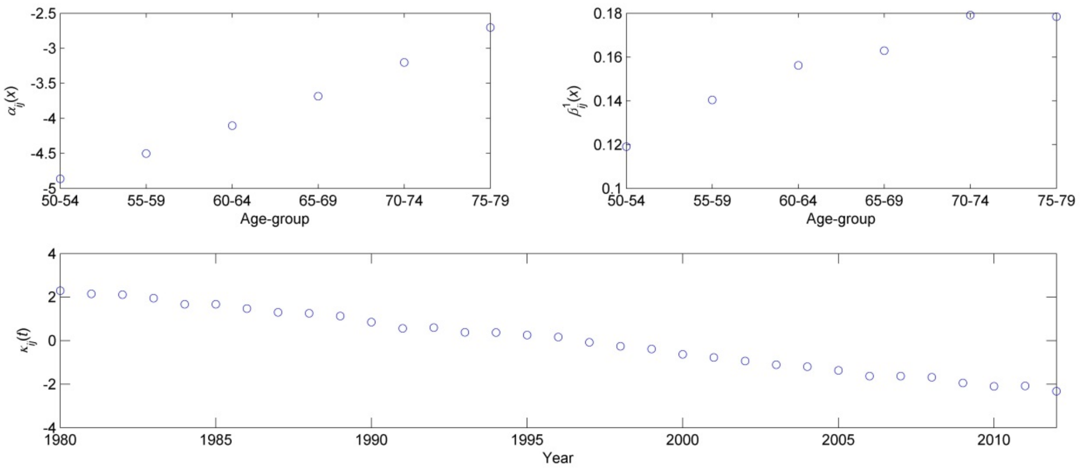
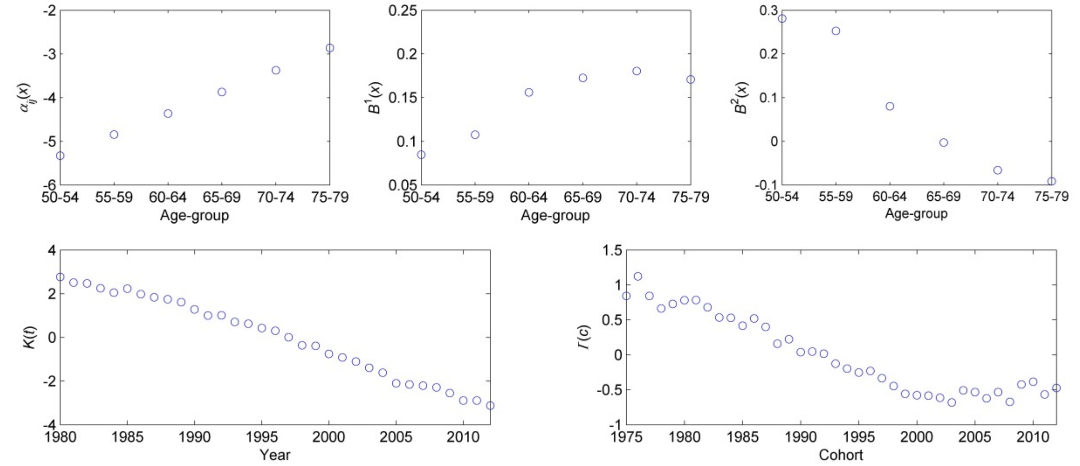


Figure A4 Parameter estimates in mortality heterogeneity, LL

Australia



New Zealand

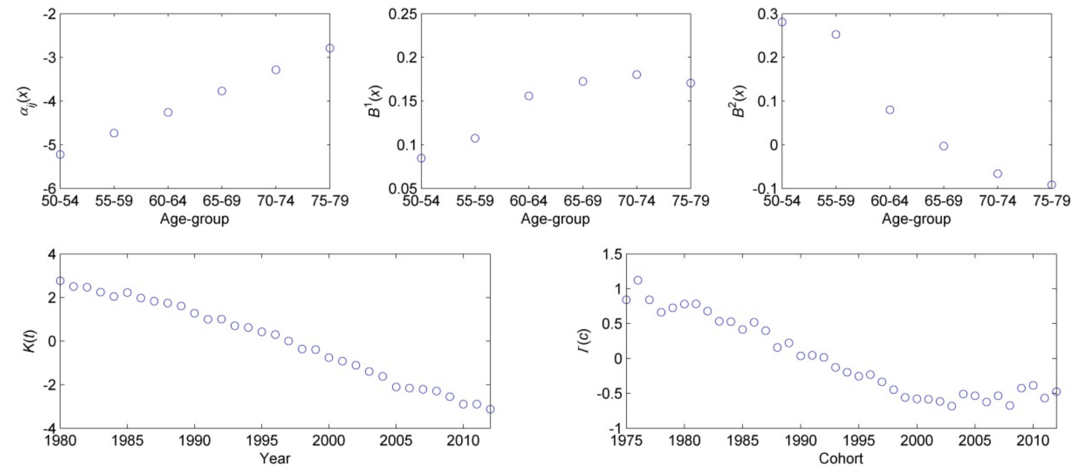
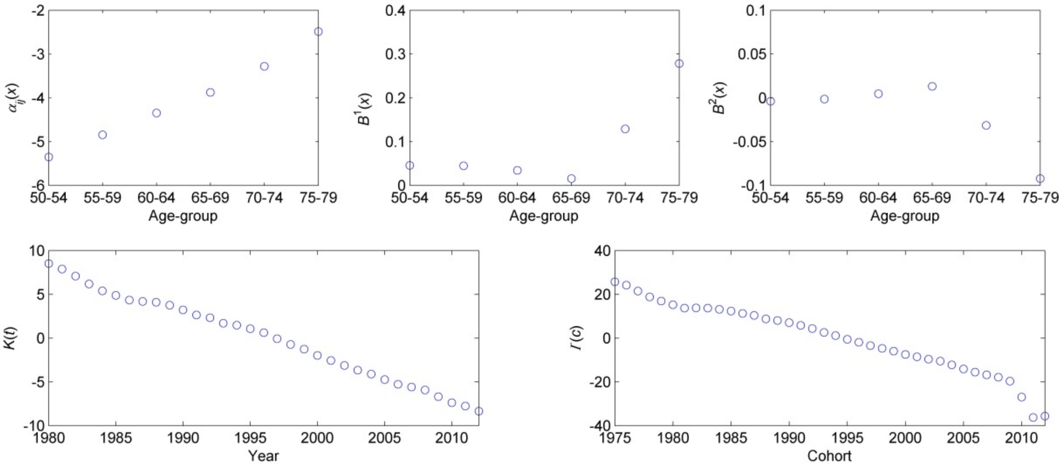


Figure A5 Parameter estimates in mortality homogeneity, RHC

Australia



Taiwan

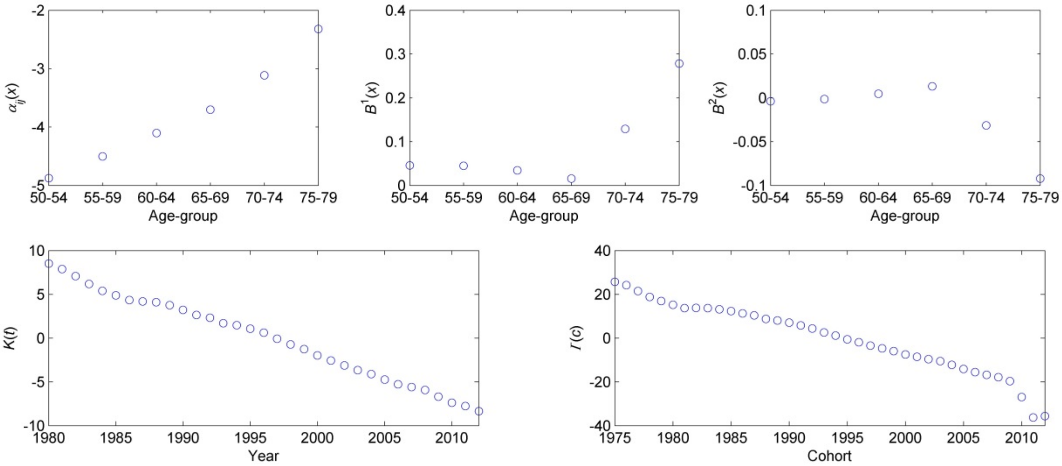


Figure A6 Parameter estimates in mortality heterogeneity, RHC

Appendix B: Mortality Projections

The coherent mortality forecast can be captured by modeling the values of $\hat{\kappa}_{ij}(t)$, $\hat{\gamma}_{ij}(c)$, $\widehat{K}(t)$, and $\hat{\Gamma}(c)$ as a time series. Given the parameter estimates of $\hat{\alpha}_{ij}(x)$, $\hat{\beta}_{ij}^{1}(x)$, $\hat{\beta}_{ij}^{2}(x)$, $\hat{B}^{1}(x)$, $\hat{B}^{2}(x)$, $\hat{\kappa}_{ij}(t)$, $\hat{\gamma}_{ij}(c)$, $\widehat{K}(t)$, and $\hat{\Gamma}(c)$, we can forecast future age-specific mortality rates. We assume that the force of mortality due to age remains uniform over each year, cohort periods of integer age, and each calendar year. Thus, our prediction captures the mortality improvement rates ($\hat{\kappa}_{ij}(t)$, $\hat{\gamma}_{ij}(c)$, $\widehat{K}(t)$, and $\hat{\Gamma}(c)$) among different age groups. Then, the autoregressive integrated moving average (ARIMA) process can be used to model the values of $\hat{\kappa}_{ij}(t), \hat{\gamma}_{ij}(c)$, $\widehat{K}(t)$, and $\hat{\Gamma}(c)$. This method is common for projecting the period and cohort effects under single and coherent models (see Lee and Carter, 1992; Renshaw and Haberman, 2003; Li and Lee, 2005; Koissi et al., 2006).

In addition, we conduct a 50-year simulation, with 50,000 simulation paths, and we use the ARIMA model to fit the period and cohort effects. Because more parameters usually reduce fit errors, we rely on the Akaike information criterion (AIC) and (Schwarz) Bayesian information criterion (BIC) for accuracy comparisons. These criteria address the likelihood function and number of parameters and frequently inform model selection. The model with the smallest AIC and BIC values then is selected. We use first-order differences for all series to create stationary series. The appropriately fitting ARIMA process results are in Table B1.

Table B1 Candidate the period and cohort effect specification, AIC and BIC

				ARMA(*p*, *q*)								
	Model	Parameter	Country	(0, 0)	(0, 1)	(0, 2)	(1, 0)	(1, 1)	(1, 2)	(2, 0)	(2, 1)	(2, 2)
AIC	LC	$\hat{\kappa}_{ij}(t)$	Australia	-30.60	-34.94	-32.97	-33.15	-32.95	-32.41	-33.74	-31.74	-29.79
			New Zealand	-17.60	-19.62	-17.65	-18.92	-17.65	-18.28	-17.64	-19.14	-24.50
			Taiwan	-42.36	-42.94	-41.29	-42.25	-42.85	-40.71	-41.84	-40.21	-45.72
	LL	$\widehat{K}(t)$	Homogeneity	-33.69	-35.88	-33.88	-34.98	-33.88	-38.04	-34.26	-40.20	-48.24
			Heterogeneity	-56.88	-54.88	-65.26	-58.11	-64.25	-63.26	-62.38	-60.38	-64.88
	RH	$\hat{\kappa}_{ij}(t)$	Australia	-20.63	-18.64	-17.22	-18.65	-16.68	-20.22	-17.21	-17.90	-23.51
			New Zealand	-17.98	-20.48	-18.66	-19.47	-18.70	-18.34	-18.17	-16.73	-23.89
			Taiwan	-32.84	-31.52	-29.52	-31.50	-29.52	-27.55	-29.54	-27.56	-29.52

		$\hat{\gamma}_{ij}(c)$	Australia	-2.96	-10.81	-13.42	-16.59	-16.88	-14.90	-16.90	-14.92	-12.95
			New Zealand	29.46	22.95	24.93	25.14	24.91	26.35	25.35	26.80	28.77
			Taiwan	-31.88	-35.46	-34.58	-36.95	-35.06	-33.26	-35.10	-33.15	-32.07
	RHC	$\hat{K}(t)$	Homogeneity	-32.06	-32.43	-30.43	-32.11	-30.43	-33.71	-30.57	-35.52	-41.56
			Heterogeneity	-12.76	-16.93	-15.44	-17.10	-15.37	-13.56	-15.44	-17.83	-16.26
		$\hat{\Gamma}(c)$	Homogeneity	4.81	4.85	1.13	4.11	6.07	2.18	5.94	4.37	3.53
			Heterogeneity	149.25	133.61	126.43	146.75	135.61	123.37	131.30	127.10	128.56
BIC	LC	$\hat{\kappa}_{ij}(t)$	Australia	-27.67	**-30.54**	-27.11	-28.75	-27.09	-25.09	-27.88	-24.41	-21.00
			New Zealand	-14.67	-15.22	-11.78	-14.53	-11.78	-10.95	-11.77	-11.81	**-15.71**
			Taiwan	**-39.43**	-38.54	-35.43	-37.85	-36.99	-33.38	-35.98	-32.89	-36.93
	LL	$\hat{K}(t)$	Homogeneity	-30.76	-31.48	-28.02	-30.59	-28.02	-30.71	-28.40	-32.87	**-39.44**
			Heterogeneity	-53.95	-50.48	**-59.39**	-53.71	-58.39	-55.93	-56.51	-53.05	-56.08
	RH	$\hat{\kappa}_{ij}(t)$	Australia	**-17.70**	-14.25	-11.36	-14.25	-10.82	-12.89	-11.34	-10.57	-14.71
			New Zealand	-15.04	**-16.09**	-12.79	-15.08	-12.83	-11.01	-12.31	-9.40	-15.09
			Taiwan	**-29.91**	-27.12	-23.66	-27.11	-23.66	-20.23	-23.68	-20.23	-20.72
		$\hat{\gamma}_{ij}(c)$	Australia	0.26	-5.97	-6.97	**-11.75**	-10.44	-6.85	-10.46	-6.87	-3.28
			New Zealand	32.68	**27.78**	31.37	29.98	31.35	34.40	31.79	34.85	38.44
			Taiwan	-28.65	-30.63	-28.14	**-32.12**	-28.62	-25.21	-28.66	-25.10	-22.41
	RHC	$\hat{K}(t)$	Homogeneity	-20.23	-17.41	-32.17	-18.47	-22.78	**-40.53**	-37.15	-34.41	-37.86
			Heterogeneity	-9.83	-12.53	-9.58	**-12.71**	-9.51	-6.23	-9.57	-10.50	-7.47
		$\hat{\Gamma}(c)$	Homogeneity	8.03	9.68	**7.57**	8.94	12.51	10.24	12.39	12.42	13.20
			Heterogeneity	152.47	138.45	132.87	151.59	142.05	**131.42**	137.74	135.16	138.23

Notes: Values in bold indicate a better fit.

Appendix C: Numerical Values for the Hedging Strategies

The numerical values for the hedging strategy of pooling life insurance and annuity products are in Table C1.

Table C1 Pooling life insurance and annuity products

Reinsurer	Hedging strategy	Model	Mean	S. D.	Skewness	Kurtosis	VaR(90)	CTE(90)
C	I	RH	0.082	0.545	0.028	3.000	-0.613	-0.869
		RHC	0.057	0.345	-0.026	2.966	-0.386	-0.549
	II	RH	1.955	0.457	-0.153	3.026	1.363	1.125
		RHC	2.156	0.324	-0.169	3.012	1.734	1.568
	III	RH	-1.791	0.649	0.146	3.024	-2.611	-2.893
		RHC	-2.041	0.375	0.091	2.974	-2.519	-2.684
D	I	RH	-0.473	0.737	-0.127	3.016	-1.426	-1.799
		RHC	0.145	1.732	-0.433	3.132	-2.161	-3.148
	II	RH	1.067	1.036	-0.197	2.980	-0.287	-0.819
		RHC	1.356	2.069	-0.866	3.817	-1.496	-2.881
	III	RH	-2.013	0.635	0.120	3.048	-2.815	-3.097
		RHC	-1.065	1.631	0.133	2.595	-3.145	-3.763

Table C2 Pooling life insurance and annuity products or annuity products across countries in RHC

Mortality characteristics	Hedging strategy	Pooling products	Mean	S. D.	Skewness	Kurtosis	VaR(90)	CTE(90)
Homogeneous	I	Annuity	-3.464	0.016	2.415	12.213	-3.477	-3.478
		Life	0.057	0.345	-0.026	2.966	-0.386	-0.549
	II	Annuity	-3.449	0.237	0.241	3.050	-3.747	-3.842
		Life	2.156	0.324	-0.169	3.012	1.734	1.568
	III	Annuity	-3.478	0.251	0.196	3.009	-3.796	-3.898
		Life	-2.041	0.375	0.091	2.974	-2.519	-2.684
Heterogeneous	I	Annuity	-1.904	0.882	0.054	2.895	-3.043	-3.430
		Life	0.145	1.732	-0.433	3.132	-2.161	-3.148
	II	Annuity	-1.885	1.251	0.29	2.621	-3.461	-3.862
		Life	1.356	2.069	-0.866	3.817	-1.496	-2.881
	III	Annuity	-1.923	1.244	0.184	2.540	-3.524	-3.947
		Life	-1.065	1.631	0.133	2.595	-3.145	-3.763

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