

# Overview of Mortality Modelling

General Guidance to  
MSc in Actuarial Science Projects

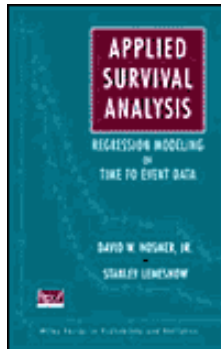
by

Richard Verrall and Zoltan Butt

# Outline

- Recommended Textbooks and Readings
- Suggested Research Topics
- Online Data Resources

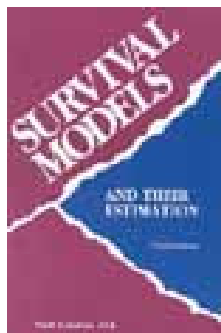
# Recommended Textbooks



- Hosmer, D.W. and Lemeshow, S. (1999) Applied survival analysis: Regression modeling of time to event data



- Elandt-Johnson, R.C. and N.L. Johnson (1999) Survival models and data analysis. Wiley Classics Library.



- London, D. (1997) Survival models and their estimation. Actex Publications. 3<sup>rd</sup> Edition.

# Suggested Research Topics

1. Mortality Rates Graduation and Forecasting (applications)
2. Laws of Mortality (literature review)
3. Longevity Issues (literature review)
4. Applications to Insurance

# 1. Mortality Rates Graduation and Forecasting

- Models and Methods of Graduation
  - Life Tables;
  - Parametric or semi-parametric methods.
- Mortality Rates Comparison
  - Application or literature review;
  - Cross-country, race, gender, etc.
- Mortality Rates by Causes of Deaths
  - Modelling (Multiple Decrement Tables, Cox PH model);
  - Implications on Health Insurance;
  - Historical changes/evolution in different countries.

# Applications of Life Tables

## Single and Multiple Decrement

- Does the data violate the assumptions?
  - Lifetimes are independent and identically distributed and are a representative sample of the population (random sampling);
  - Censoring is independent (non-informative).
- Clues to look out for:
  - Patterns in the plot of data against time;
  - The effect of strong explanatory variable(s) (measured or not);
  - Patterns in censoring times against potential factors (not necessary part of the data).
  - Effect of grouping algorithm.

# Recommended Readings

- London, D. (1985) *Graduation: The revision of Estimates*. Actex Publications.
- Benjamin, B. and Pollard, J.H. (1993) *The Analysis of Mortality and Other Actuarial Statistics*, The Institute of Actuaries and Faculty of Actuaries.
- Brown, R.L. (1997) *Introduction to the Mathematics of Demography*, 3<sup>rd</sup> Edition, ACTEX Publications.
- Willets, R.C. (2004) “The cohort effect: Insights and explanations”. Paper presented to the IFA.  
(downloadable from: <http://www.actuaries.org.uk/> )
- Forfar, D.O., McCutcheon, J.J. and Wilkie, A.D. (1988) “On graduation by mathematical formula”, *Journal of the Institute of Actuaries*, **vol. 115**, pp. 1–149.
- Renshaw, A.E. (1991) “Actuarial graduation practice and generalized linear and non-linear models”, *Journal of the Institute of Actuaries*, **vol. 118**, pp. 295–312.

## 2. Laws of Mortality (literature review)

- Historical concepts and theories;
- Senescence and ageing processes;
- Lifetime distributions and their estimation by age, duration;



# Historical Notes

- Earliest ‘population’ mortality records collected in the late 16<sup>th</sup> century by local parishes.
- In 1662 Graunt carries out the first detailed investigation of mortality of Londoners.
- In 1693 Halley constructs the first ‘life table’ of Breslau.
- In 1746 Deparcieux uses a life table method to show increases in life expectancies.
- In 1766 Bernoulli first studies the properties of mortality intensity  $\mu(x)$ . Lambert in 1772 calls the inverse of this the ‘force of vitality’ and develops the earliest mathematical models for the survivor function  $S(x)$ .

# Historical Notes (continues)

## Mortality Models

- In 1725 de Moivre first proposed a piecewise linear survivor function  $S(x) - S(x+t) = a t$ .
- In 1772 Lambert first expresses the gradual 'exhaustion of man's power' in a mathematical formula.
- In 1798 Malthus predicted exponential increase of human population with consequent misery and famine unless family size was regulated. The model was later adjusted by Verhulst in 1838, to limit the population growth predictions.
- In 1825 Gompertz proposed a survivor function which results in an exponentially rising mortality intensity. This model was later extended by Makeham in 1860 to allow for young mortality.
- In 1872 Thiele develops the first complex model that attempts to accurately model  $\mu(x)$  over the whole life span.

# Recommended Readings

- Gompertz, B. (1872) “On one uniform law of mortality from birth to extreme old age, and on the law of sickness”, *Journal of the Institute of Actuaries*, **vol. 16**, pp. 329–344.
- Makeham, W.M. (1867) “On the law of mortality”, *Journal of the Institute of Actuaries*, vol. 13, pp. 325–358.
- Olshansky, S.J. and Carnes, B.A. (1997) “Ever since Gompertz”, *Demography*, **vol. 34**, Issue I, pp. 1–6.
- Willemse, W.J. (2001) *Computational Intelligence: Mortality Models for the actuary*. PhD Dissertation. Delft University Press, pp. 13–22.

# Modern Theories of Mortality and Senescence

- Biodemography
  - Proposes biological explanations for the differences between life spans of different species;
  - Studies cellular processes and genetical effects that might influence longevity.
- Evolutionary Theory of Senescence
  - Hypothesises that evolutionary forces and adaptation strategies shape the rate of ageing of different species;
  - Argues for a dynamic (i.e. changing) upper limit to the life span of a species, determined by current evolutionary conditions.
- Reliability Theory of Ageing
  - Statistical models that essentially predict the rate of erosion of complex systems through component failures;
  - It attempts to incorporate specific biological knowledge into mathematical models of mortality.

# Recommended Readings

- Carnes, B.A., Olshansky, S.J. and Grahn, D. (1996) “Continuing the search for the law of mortality”, *Population and Development Review*, **vol. 22**, Issue 2, pp. 231–264.
- Held, G. (2002) “Research into the aging process: a survey”, *North American Actuarial Journal*, **vol. 6**, no. 3, pp. 30–37.
- Le Bourg, É. (2001) “A mini-review of the evolutionary theories of aging. Is it the time to accept them?” , *Demographic Research*, **vol. 4**, article 1, pp. 1–28.
- Olshansky, S.J. and Carnes, B.A. (1997) “Ever since Gompertz”, *Demography*, **vol. 34**, Issue I, pp. 6–15.
- Gavrilov, L.A. and Gavrilova, N.S. (2002) “Biodemographic (reliability) theory of aging and longevity”, Paper presented at The 2002 Annual Meeting of the Population Association of America. Atlanta. Session 135. pp. 1-53.

### 3. Longevity Issues (literature review)

- Mortality rates at very old ages,
  - Frailty models (Unobserved Heterogeneity):
  - Life expectancy estimation, projections
  - historical changes;
- Implications of the Ageing Population
  - Pensions; Social Planning; Long Term Care; etc

# Recommended Readings

- Willets, R.C. (2004) “Longevity in the 21<sup>st</sup> century”. Paper presented to the IFA.
- IFA Discussion Meeting: “Living longer is bad for the nation’s wealth”. October 2003.  
(both are downloadable from: <http://www.actuaries.org.uk/> )
- Vaupel, J.W. (1998) “Demographic analysis of aging and longevity”, *The American Economic Review*, **vol. 88**, Issue 2, Papers and Proceedings of the 110<sup>th</sup> Annual Meeting of the American Economic Association, pp. 242–247.
- Yashin, A.I. and Iachine, I.A. (1997) “How frailty models can be used for evaluating longevity limits: Taking advantage of an interdisciplinary approach”, *Demography*, **vol. 34**, pp. 31–48.

# Frailty Models

- These models assume that (unobserved) individual differences in mortality determine the (observed) overall population mortality.
- These models are often referred to as *random effects* models, given that they allow for random (unobserved) mortality factors that influence the mortality of subgroups of populations.
- As a result, selection effects operate in the modelled population, so that more frail members are removed earlier and at increasing age the population is likely to be made up of stronger individuals, which would explain the slowing rate of mortality at older ages.
- In essence, when a population is made up of non-homogeneous individuals, then the individual mortality characteristics are measurable only if the compositional dynamics of the population are known.



# Recommended Readings

- Aalen, O.O. (1988) “Heterogeneity in survival analysis”, *Statistics in Medicine*, **vol. 7**, pp. 1121–1137.
- Congdon, P. (1994) “Analyzing mortality in London: Life Tables with frailty”, *Journal of the Royal Statistical Society, Series D*, **vol. 43**, pp. 277–308.
- Hougaard, P. (1984) “Life Table methods for heterogeneous populations: Distributions describing heterogeneity”, *Biometrika*, **vol. 71**, pp. 75–83.
- Manton, K.G., Stallard, E. and Vaupel, J. W. (1981) “Methods for comparing the mortality experience of heterogeneous populations”, *Demography*, **vol. 18**, pp. 389–410.
- Vaupel, J. W., Manton, K.G. and Stallard, E. (1979) “The impact of heterogeneity in individual frailty on the dynamics of mortality”, *Demography*, **vol. 16**, pp. 439–454.

## 4. Applications to Insurance

- Life Insurance, Pensions, Social policy planning;
- Disability Insurance, Income Protection Insurance and Long Term Care;
- NPV estimation (due to morbidity) or pricing of the insurance schemes;

# Recommended Readings

- Sithole, T., Haberman, S. and Verrall, R.J. (2000) “An investigation into parametric models for mortality projections, with applications to immediate annuitants and life office pensioners”, *Insurance: Mathematics and Economics*, **vol. 27**, pp. 285–312.
- Vinsonhaler, C., Ravishanker, N., Vadiveloo, J. and Rasoanaivo, G. (2001) “Multivariate analysis of pension plan mortality data”, *North American Actuarial Journal*, **vol. 5** (2), pp. 126–138.
- Chenl, Y-P. (2002) “Funding Long Term Care”, Paper presented to the Society of Actuaries.
- Czado, C. and Rudolph, F. (2002) “Applications of survival analysis methods to Long Term Care Insurance”, *Insurance: Mathematics and Economics*, **vol. 31**, pp. 395–413.
- Jones, B.L. (1997) “Methods for the analysis of CCRC Data”, *North American Actuarial Journal*, **vol 1** (2), pp. 40–54.

# Data Resources for Applications United Kingdom (England and Wales)

- Office for National Statistics Online:  
<http://www.statistics.gov.uk/> (Search: mortality)
- Government Actuary's Department (GAD):  
[http://www.gad.gov.uk/Life\\_Tables/Interim\\_life\\_tables.htm](http://www.gad.gov.uk/Life_Tables/Interim_life_tables.htm)
- Institute of Actuaries and Faculty of Actuaries:
  - Continuous Mortality Investigation Bureau;  
<http://www.actuaries.org.uk/> and follow the links:  
Resource centre → CMI Reports → CMI Working Papers.
- UK Data Archive:  
<http://www.data-archive.ac.uk/>

# Data Resources (continues)

## USA and International

- National Cancer Institute (NCI):  
<http://seer.cancer.gov> contains:
  - Surveillance Epidemiology and End Results Program  
SEER 1973-2001 Public-Use Data of USA;
  - USA mortality and population data;
- National Center for Health Statistics (NCHS):  
<http://www.cdc.gov/nchs/> contains:
  - USA socio-economic data and reports.
- World Health Organization Statistical Information System (WHOSIS):  
<http://www3.who.int/whosis/menu.cfm> or  
<http://www.who.int/research/en/> and follow link:  
WHO mortality database.
- Other countries national statistics and mortality data is usually available through their official actuarial sites.