Statistical Computing and Simulation

Spring 2026

Assignment 4, Due May 15/2026

1. Mortality Compression was proposed by Fries in 1980 and it is associated to Rectangularization of the survival curve, or the majority of deaths would occur in a short range of ages. Cheung et al. (2005) conjectured that the distribution of number of deaths (i.e., d_x) looks like normal distribution, at least for the groups older than the modal age. Try at least three different methods to estimate modal and SD for Taiwan's 2022-2024 death records.

2. Consider a multinomial observation $X = (x_1, x_2, x_3, x_4)$ with class probabilities given by $(p_1, p_2, p_3, p_4) = (0.5-\theta, 0.3-\theta, 0.1+\theta, 0.1+\theta)$, where $0 < \theta < 1$. The sample size is $n = \sum x_i$ and the parameter θ is to be estimated from the observed frequencies (1408, 640, 961, 991), i.e., sample size 4,000. Use the secant, Ridder's (or Brent's), and Newton-Raphosn methods to find the MLE (via $l'(\theta)$). You may choose your own starting points and convergence criterion (preferred 10^{-6}or smaller).

3. Evaluate the following quantities by both numerical and Monte Carlo integration, and compare their errors with respect to the numbers of observations used. Also, propose at least two simulation methods to reduce the variance of Monte Carlo integration and compare their variances.

   (a) $\theta = \int_0^1 sin^2(1/x)dx$

   (b) The volume of $x^2 + y^2/4 + z^2/9 \le 1$ in the unit cubic (0,1) × (0,1) × (0,1).

4. Evaluate the CDF of standard normal distribution *Φ(x)* using the method of Important Sampling and other Variance Reduction Methods (at least two different methods). Consider *x* = –6, –5, –4, –3.5, –3, –2.5, –2.

5. Let (X, Y) be a bivariate random variable following normal distribution, with mean (μ_x, μ_y) and variance matrix $\begin{pmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{pmatrix}$. Using Monte Carlo simulation to estimate the probability of $P(X+Y<k)$, where $k = 0, 1, 2, 3, 4$, $\mu_x = \mu_y = 0 = \rho$, and $\sigma_x = \sigma_y = 1$. In addition, propose at least three variance methods and compare their results with those using Monte Carlo simulation. Also,

redo the preceding simulation if the correlation coefficient is –0.9, –0.5, 0.3, or 0.7.

6. Experiment with as many variance reduction techniques as you can think of to apply the problem of evaluating $P(X > 1)$ for $X \sim$ Cauchy.