

Statistical Computing and Simulation

Spring 2026

Assignment 3, Due April 28/2026

1. To compare teaching, twenty schoolchildren were divided into two groups: ten taught by conventional methods and ten taught by an entirely new approach. The following are the test results:

Conventional	65	79	90	75	61	85	98	80	97	75
New	90	98	73	79	84	81	98	90	83	88

Are the two teaching methods equivalent in result? You need to use permutation test, (parametric and non-parametric) bootstrap, and parametric test, and then compare their differences in testing.

2. (a) Write a small program to perform the “Permutation test” and test your result on the correlation of DDT vs. eggshell thickness in class, and the following data:

X	585	1002	472	493	408	690	291
Y	0.1	0.2	0.5	1.0	1.5	2.0	3.0

Check your answer with other correlation tests, such as regular Pearson and Spearman correlation coefficients.

- (b) Simulate a set of two correlated normal distribution variables, with zero mean and variance 1. Let the correlation coefficient be 0.2 and 0.8. (Use Cholesky!) Then convert the data back to Uniform(0,1) and record only the first decimal number. (亦即只取小數第一位，0至9的整數) Suppose the sample size is 10. Apply the permutation test, Pearson and Spearman correlation coefficients, and records the p-values of these three methods. (10,000 simulation runs)
3. Using simulation to construct critical values of the Mann-Whitney-Wilcoxon test in the case that $2 \leq n_1, n_2 \leq 10$, where n_1 and n_2 are the number of observations in two populations. (Note: The number of replications shall be at least 10,000.)
4. Similar to what Efron did in the Law school data example, compute the bootstrap simulation for 50, 100, ..., 10,000 replications. But, instead of using the original 15 observations, we want to know if the number of observations plays an important role. Randomly select 10, 15, 20, and 25 observations and then see if the bootstrap variance converges as the number of replications increases. (Note: You also need to compare your results with that of population.)

school	LSAT	GPA	school	LSAT	GPA	school	LSAT	GPA	school	LSAT	GPA
1	622	3.23	22	614	3.19	43	573	2.85	63	572	3.08
2	542	2.83	23	628	3.03	44	644	3.38	64	610	3.13
3	579	3.24	24	575	3.01	(45)	545	2.76	65	562	3.01
(4)	653	3.12	25	662	3.39	46	645	3.27	66	635	3.30
5	606	3.09	26	627	3.41	(47)	651	3.36	67	614	3.15
(6)	576	3.39	27	608	3.04	48	562	3.19	68	546	2.82
7	620	3.10	28	632	3.29	49	609	3.17	69	598	3.20
8	615	3.40	29	587	3.16	(50)	555	3.00	(70)	666	3.44
9	553	2.97	30	581	3.17	51	586	3.11	71	570	3.01
10	607	2.91	(31)	605	3.13	(52)	580	3.07	72	570	2.92
11	558	3.11	32	704	3.36	(53)	594	2.96	73	605	3.45
12	596	3.24	33	477	2.57	54	594	3.05	74	565	3.15
(13)	635	3.30	34	591	3.02	55	560	2.93	75	686	3.50
14	581	3.22	(35)	578	3.03	56	641	3.28	76	608	3.16
(15)	661	3.43	(36)	572	2.88	57	512	3.01	77	595	3.19
16	547	2.91	37	615	3.37	58	631	3.21	78	590	3.15
17	599	3.23	38	606	3.20	59	597	3.32	(79)	558	2.81
18	646	3.47	39	603	3.23	60	621	3.24	80	611	3.16
19	622	3.15	40	535	2.98	61	617	3.03	81	564	3.02
20	611	3.33	41	595	3.11	62	637	3.33	(82)	575	2.74
21	546	2.99	42	575	2.92						

Sampled schools have bold-faced school numbers.

- Suppose the central death rate $m_x = 0.05$. Compute the mortality rate q_x under the uniform death distribution, constant force, and hyperbolic assumption.
- This problem is to solve linear optimization (or linear programming). Use the package “lpSolve” to maximize profits = $0.15X_1 + 0.40X_2$, given that

$$0.2X_1 + 0.7X_2 \leq 100$$

$$X_1 + X_2 \leq 200$$

$$\text{and } X_1 \geq 0, X_2 \geq 0.$$