Statistical Computing and Simulation

Spring 2026

Assignment 1, Due March 20/2026

1. (a) Consider the combination of 3 multiplicative congruential generators, i.e.,

$$u_i = \frac{x_i}{30269} + \frac{y_i}{30307} + \frac{z_i}{30323} \pmod 1$$

with $x_i = 171\, x_{i-1} \pmod{30269}$, $y_i = 172\, y_{i-1} \pmod{30307}$, $z_i = 170\, z_{i-1} \pmod{30323}$.

(b) Hand calculators use $U_{n+1} = (\pi + U_n)^5 \pmod 1$ to generate random numbers between 0 and 1. Compare the result with those in (a) & (b), and discuss your finding based on the comparison. (Note: You must notify the initial seed number used, and you may adapt 0.05 as the α value. Also, you may find warning messages for conducting the Goodness-of-fit test, and comment on the Goodness-of-fit test.)

2. (a) In class, we often use simulation tools in R, e.g., "sample" or "ceiling(runif)," to generate random numbers from 1 to k, where k is a natural number. Using graphical tools (such as histogram) and statistical tests to check which one is a better tool in producing uniform numbers between 1 and k. (Note: You may check if the size of k matters by, for example, assigning k a small or big value.)

(b) Fibonacci numbers, defined as $X_{n+1} = X_n + X_{n-m}\ (mod\ \ 1)$, is another way of generating random numbers. The usual setting is letting $m = 1$ and see if (X_n) is a sequence of random numbers from *U(0,1)*. However,$x_n < x_{n+1} < x_{n-1}$ and$x_{n-1} < x_{n+1} < x_n$never appear under this setting. In general, the performances of Fibonacci numbers would be close to "random" as *m* increases. Write a program to generate Fibonacci numbers and test if they are "good" random numbers given varies choices of *m*.

3. There are several ways for acquiring random numbers, e.g., using irrational numbers.

(a) Find at least two ways to download the decimal places of irrational numbers (such as $\sqrt{2}$) in R or other software?

(b) Check via graphic tools if the first 5,000 digits after the decimal point of $\sqrt{2}$ violate the assumption of random numbers.

4. Write your own R programs to perform Gap test, Permutation test, and run test. Then use this program to test if the uniform random numbers generated from Minitab (or

SAS, SPSS, Excel) and R are independent.

5. Check if the first 10,000 digits after the decimal point of $\sqrt{2}$ satisfy the assumption of random numbers. (Note: You may choose a single digit or two or more digits as the unit of calculation.)
6. The following table shows the winning numbers of first 20 Taiwan Lottery (starting in 2002), which picks 6 numbers from 42 balls plus a "Power Ball." Go to the web page of Taiwan Lottery and download the winning numbers of 2025 Grand Lotto (大樂透). Choose your tools to check whether these winning numbers are random.

Date	Winning Numbers						Power Ball	Date	Winning Numbers						Power Ball
0329	22	31	34	25	21	19	13	0222	32	10	15	02	30	23	36
0326	05	18	25	26	35	42	29	0219	24	20	36	19	07	12	26
0321	32	21	09	27	31	06	2	0215	01	06	07	12	42	20	35
0319	05	25	02	16	32	09	7	0212	25	39	20	38	29	37	28
0315	15	29	05	36	13	10	1	0208	26	02	15	29	04	33	39
0312	36	16	12	26	08	34	5	0205	17	39	03	15	11	01	34
0308	04	40	27	21	14	05	12	0201	13	39	28	30	25	29	21
0305	29	04	10	23	39	14	36	0129	07	09	29	34	39	36	16
0301	30	12	40	32	35	20	34	0125	28	31	16	35	06	30	2
0226	40	06	20	29	38	35	41	0122	10	32	13	04	09	33	37