Statistical Computing and Simulation

Spring 2025

Assignment 2, Due March 28/2024

1. We can use the command "arima.sim" in R to generate random numbers from ARIMA models.
   (a) We generate 100 random numbers from AR(2) with parameter values $(\phi_1, \phi_2) = (\theta, \theta)$ and apply correlation between x_i vs. x_{i+1} and x_i vs. x_{i+2} as a tool for verifying independence. You should repeat the simulation at least 1,000 times and try different θ values, such as θ = 0, 0.05, 0.10, 0.15, and 0.20.
   (b) Using ARIMA random numbers to evaluate the type-1 and type-2 errors of various independence tests, e.g., Gap, Up-and-down, and Permutation tests.
2. Describe an algorithm for generating from multinomial distribution

$$f(x_1, x_2, \ldots, x_k) = \frac{n!}{x_1! x_2! \cdots x_k!} p_1^{x_1} p_2^{x_2} \cdots p_k^{x_k},$$

   where $\sum_{i=1}^{k} p_i = 1$ and $\sum_{i=1}^{k} x_i = n$. (Note: Searching on the web, see if there are better ways for generating random numbers from multinomial distribution.)
3. For uniform (0,1) random variables $U_1, U_2, \ldots,$ define $N = \min\{n : \sum_{i=1}^{n} U_i > 1\}$. That is, N is the number of random numbers that must be summed to exceed 1
   (a) Estimate $E(N)$ with standard errors by generating 1,000, 2,000, 5,000, 10,000, and 100,000 values of N, and check if the pattern in the estimated value and its s.e.
   (b) Compute the density function of N, $E(N)$, and $Var(N)$.

4. Propose an algorithm for generating from normal distribution via rejection method and you need to check the goodness-of-fit and independence. (You only need to choose one test for testing independence.)

5. Given the following matrix:

$$A = \begin{bmatrix} 1 & 0.5 & 0.25 & 0.125 \\ 0.5 & 1 & 0.5 & 0.25 \\ 0.25 & 0.5 & 1 & 0.5 \\ 0.125 & 0.25 & 0.5 & 1 \end{bmatrix}.$$

(a) Write a program to compute the Cholesky decomposition of A. To double check your result, use the command “chol” in R to verify the result.

(b) Use the commands “eigen”, “qr”, and “svd” on A and check if these commands work properly.

6. Figure a way to find the parameters of AR(1) and AR(2) models for the data “lynx” in R. Also, apply statistical software (e.g., R, SAS, SPSS, & Minitab) to get estimates for the AR(1) and AR(2) model and compare them to those from your program.