Statistical Computing and Simulation

Spring 2024

Assignment 4, Due May 17/2024

1. Suppose the central death rate $m_x = 0.04$. Compute the mortality rate q_x under the uniform death distribution, constant force, and hyperbolic assumption.

2. Try at least three different methods to find the estimates of B and C for the Gompertz model, $\mu_x = BC^x,\ x > 0,$ using the Taiwan data in 2019-2021. You may count "nlminb", "nls" or "opt" as one of the methods (for replacing Newton's method). Also, similar to what we saw in the class, discuss the influence of starting points to the number of iterations. You may choose the male data or female data.

3. Consider a multinomial observation $X = (x_1, x_2, x_3, x_4)$ with class probabilities given by $(p_1, p_2, p_3, p_4) = (\frac{2+\theta}{4}, \frac{1-\theta}{4}, \frac{1-\theta}{4}, \frac{\theta}{4})$, where $0 < \theta < 1.$ The sample size is $n = \sum x_i$ and the parameter θ is to be estimated from the observed frequencies (1997, 906, 904, 32), i.e., sample size 3839. Use the secant, Ridder's (or Brent's), and Newton-Raphosn methods to find the MLE (via $l'(\theta)$). You may choose your own starting points and convergence criterion (preferred 10^{-6} or smaller).

4. Evaluate the CDF of standard normal distribution $\Phi(x)$ using the method of Important Sampling and other Variance Reduction Methods (at least two different methods). Consider $x = -6, -5, -4, -3.5, -3, -2.5, -2$.

5. Evaluate the following quantity by both numerical and Monte Carlo integration, and compare their errors with respect to the numbers of observations used. Also, propose at least two simulation methods to reduce the variance of Monte Carlo integration and compare their variances.

$$\theta = \int_0^1 e^{x^2} dx$$

6. Let $X_i,\ i = 1, \ldots, 5$ be random variables, following the exponential distribution with mean 1. Consider the quantity θ defined by $\theta = P\{\sum_{i=1}^{5} iX_i \geq 21.6\}$. Propose at least three simulation methods to estimate θ and compare their variances.