Statistical Computing and Simulation

Spring 2024

Assignment 2, Due April 2/2024

1. We can use the command "arima.sim" in R to generate random numbers from ARIMA models.
   (a) We generate 100 random numbers from AR(2) with parameter values $(\phi_1, \phi_2) = (\theta, \theta)$ and apply correlation between x_i vs. x_{i+1} and x_i vs. x_{i+2} as a tool for verifying independence. You should repeat the simulation at least 1,000 times and try different θ values, such as θ = 0, 0.05, 0.10, 0.15, and 0.20.
   (b) Using ARIMA random numbers to evaluate the type-1 and type-2 errors of various independence tests, e.g., Gap, Up-and-down, and Permutation tests.
2. (a) Test the generation methods of normal distribution introduced in class, i.e., Box-Muller, Polar, Ratio-of-uniform, and also the random number generators from R. Based on your simulation results, choose the "best" generator.

   (b) In the class we mentioned it is found by several researchers that

   a (multiplier) = 131
   c (increment) = 0
   m (modulus) = 2^35

   would have $X \in (-3.3, 3.6)$, if plugging congruential generators into the Box-Muller method.   Verify if you would have similar results.
3. Write a program to generate random numbers from Poisson distribution. This program has the function for choosing the starting points, such as from starting from 0, mean, or median. In addition, this program can record the numbers of steps needed for generating a random number. Similar to what we saw in class, if λ = 10, compare the numbers of steps needed if starting from 0 and mean.
4. Show that the following algorithm generates random numbers from t_v-variate. (It is a rejection algorithm with $g(x) \propto \min(1, 1/x^2)$.)

   1. Generate $U, U_1 \sim U(0,1)$.
   2. If $U<1/2$ then $X = 1/(4U-1), V=x^{-2}U_1$
   else $X = 4U-3$, $V = U_1$.
   3. If $V < 1 - |X|/2$ go to 5.
   4. If $V \geq (1 + X^2/v)^{-(v+1)/2}$ go to 1.
   5. Return X.

5. Given the following matrix: $$A = \begin{bmatrix} 1 & 0.5 & 0.25 & 0.125 \\ 0.5 & 1 & 0.5 & 0.25 \\ 0.25 & 0.5 & 1 & 0.5 \\ 0.125 & 0.25 & 0.5 & 1 \end{bmatrix}.$$

   (a) Write a program to compute the Cholesky decomposition of A. To double check your result, use the command "chol" in R to verify the result.

   (b) Use the commands "eigen", "qr", and "svd" on A and check if these commands work properly.

6. Figure a way to find the parameters of AR(1) and AR(2) models for the data "lynx" in R. Also, apply statistical software (e.g., R, SAS, SPSS, & Minitab) to get estimates for the AR(1) and AR(2) model and compare them to those from your program.