Statistical Computing and Simulation

Spring 2024

Assignment 1, Due March 15/2024

1. (a) Write a computer program using the Mid-Square Method using 6 digits to generate 10,000 random numbers ranging over [0,999999]. Use the Kolmogorov-Smirnov Goodness-of-fit test to see if the random numbers that you create are uniformly distributed. (Note: You must notify the initial seed number used, and you may adapt 0.05 as the α value. Also, you may find warning messages for conducting the Goodness-of-fit test, and comment on the Goodness-of-fit test.)

   (b) Consider the combination of 3 multiplicative congruential generators, i.e.,

   $$u_i = \frac{x_i}{30269} + \frac{y_i}{30307} + \frac{z_i}{30323} \pmod 1$$

   with $x_i = 171\, x_{i-1} \pmod{30269}$, $y_i = 172\, y_{i-1} \pmod{30307}$, $z_i = 170\, z_{i-1} \pmod{30323}$.

   Compare the results in (a) and (b), and discuss your findings.

2. (a) In class, we often use simulation tools in R, e.g., "sample" or "ceiling(runif)," to generate random numbers from 1 to k, where k is a natural number. Using graphical tools (such as histogram) and statistical tests to check which one is a better tool in producing uniform numbers between 1 and k. (Hint: You may check if the size of k matters by, for example, assigning k a small or big value.)

   (b) Hand calculators often use $U_{n+1} = (\pi + U_n)^5 \pmod 1$ to generate random numbers between 0 and 1. Compare the result with those in #1, and discuss your finding based on the comparison.

3. There are several ways for checking the goodness-of-fit for empirical data. In specific, there are a lot of normality tests available in R. Generate a random sample of size 10, 50, and 100 from N(0,1) and t-distribution (with degrees 10 and 20) in R. You may treat testing random numbers from t-distribution as the power. For a level of significance α = 0.05 test, choose at least four normality tests in R ("nortest" module) to check if this sample is from N(0,1). Tests used can include the Kolmogorov-Smirnov test and the Cramer-von Mises test. Note that you need to compare the differences among the tests you choose.

4. Write your own R programs to perform Gap test, Permutation test, and run test. Then use this program to test if the uniform random numbers generated from Minitab (or SAS, SPSS, Excel) and R are independent.
5. (a) Use the search engine to download the first one million digits of pi (for example, https://www.piday.org/million/), and check via graphic tools if the numbers violate the assumption of random numbers.

   (b) Apply the appropriate tools to test if the random numbers from (a) satisfy the assumption of random numbers.
6. The following table shows the winning numbers of first 20 Taiwan Lottery (starting in 2002), which picks 6 numbers from 42 balls plus a "Power Ball." Choose your tools to check whether these winning numbers are random.

Date	Winning Numbers						Power Ball	Date	Winning Numbers						Power Ball
0329	22	31	34	25	21	19	13	0222	32	10	15	02	30	23	36
0326	05	18	25	26	35	42	29	0219	24	20	36	19	07	12	26
0321	32	21	09	27	31	06	2	0215	01	06	07	12	42	20	35
0319	05	25	02	16	32	09	7	0212	25	39	20	38	29	37	28
0315	15	29	05	36	13	10	1	0208	26	02	15	29	04	33	39
0312	36	16	12	26	08	34	5	0205	17	39	03	15	11	01	34
0308	04	40	27	21	14	05	12	0201	13	39	28	30	25	29	21
0305	29	04	10	23	39	14	36	0129	07	09	29	34	39	36	16
0301	30	12	40	32	35	20	34	0125	28	31	16	35	06	30	2
0226	40	06	20	29	38	35	41	0122	10	32	13	04	09	33	37