Statistical Computing and Simulation

Spring 2023

Assignment 2, Due March 31/2023

1. The following table shows the winning numbers of first 20 Taiwan Lottery (starting in 2002), which picks 6 numbers from 42 balls plus a "Power Ball." Choose your tools to check whether these winning numbers are random.

Date	Winning Numbers						Power Ball	Date	Winning Numbers						Power Ball
0329	22	31	34	25	21	19	13	0222	32	10	15	02	30	23	36
0326	05	18	25	26	35	42	29	0219	24	20	36	19	07	12	26
0321	32	21	09	27	31	06	2	0215	01	06	07	12	42	20	35
0319	05	25	02	16	32	09	7	0212	25	39	20	38	29	37	28
0315	15	29	05	36	13	10	1	0208	26	02	15	29	04	33	39
0312	36	16	12	26	08	34	5	0205	17	39	03	15	11	01	34
0308	04	40	27	21	14	05	12	0201	13	39	28	30	25	29	21
0305	29	04	10	23	39	14	36	0129	07	09	29	34	39	36	16
0301	30	12	40	32	35	20	34	0125	28	31	16	35	06	30	2
0226	40	06	20	29	38	35	41	0122	10	32	13	04	09	33	37

2. (a) Use the search engine to download the first one million digits of pi (for example, https://www.piday.org/million/), and check via graphic tools if the numbers violate the assumption of random numbers.

   (b) Apply the appropriate tools to test if the random numbers from (a) satisfy the assumption of random numbers.

3. $(\sum_{i=1}^{12} U_i - 6)$ can be used to approximate N(0,1) distribution, where $U_i's$ are random sample from U(0,1).
   (a) Based on $\alpha = 0.05$, compare the results of the Chi-square test and the Kolmogorov-Smirnov test, and see if there are any differences.
   (b) Design two tests of independence (which are not the same as you saw in class) and apply them on the random sample that you generate.

   Note that you need to simulate random sample of *U(0,1)* via the LCG. Also, a minimum of 10,000 simulation runs is required.

4. We can use the command "arima.sim" in R to generate random numbers from ARIMA models.

(a) We generate 100 random numbers from AR(2) with parameter values $(\phi_1, \phi_2) = (\theta, \theta)$ and apply correlation between x_i vs. x_{i+1} and x_i vs. x_{i+2} as a tool for verifying independence. Note that you should repeat the simulation fat least 1,000 times and try different θ values, such as θ = 0, 0.05, 0.10, 0.15, and 0.20.

(b) Using ARIMA random numbers to evaluate the type-1 and type-2 errors of various independence tests, e.g., Gap, Up-and-down, and Permutation tests.

5. (a) Using Rejection Method (or Inversion) described in the class, to generate random numbers from Cauchy distribution. (Note: You need to check the goodness-of-fit and independence.)

   (b) Compare with the methods of "Ratio of Uniform" (in-class) and "Ratio of Normal" and give your suggestion (such as choosing the "best" method) for creating random numbers from Cauchy distribution.

6. Write a program to check the Table method and the Alias method for generating r.v. from B(3,1/3). Also, compare the speed of generation for the two methods.

7. For uniform (0,1) random variables $U_1, U_2, \ldots$, define $N = \min\{n : \sum_{i=1}^{n} U_i > 1\}$. That is, *N* is the number of random numbers that must be summed to exceed 1

   (a) Estimate *E(N)* with standard errors by generating 1000, 2000, 5000, 10000, and 100000 values of *N*, and check if there are any patterns in the estimate and its s.e.

   (b) Compute the density function of *N*, *E(N)*, and *Var(N)*.