

Section 6.1

2. By the Archimedean Property there exists a natural number n such that $(b-a)/n < \delta$; if we define $x_i = a + i(b-a)/n$, $i = 0, \dots, n$, then $\{x_0, \dots, x_n\}$ is a partition of $[a, b]$ with $x_i - x_{i-1} = (b-a)/n < \delta$ for all $i = 1, \dots, n$, as desired.
4. Lemma 6.1 with $m = 0$ implies that $L(f, P) \geq 0$ for all partitions P of $[a, b]$; the result then follows from the definition of lower integral.
6. We let $P = \{x_0, x_1, \dots, x_n\}$, so that by our assumption and by the properties of lower and upper sums, $\inf\{f(x) \mid x \text{ in } [x_{i-1}, x_i]\} = \sup\{f(x) \mid x \text{ in } [x_{i-1}, x_i]\}$ for all i ; it follows from the definitions of infimum and supremum that $\{f(x) \mid x \text{ in } [x_{i-1}, x_i]\}$ contains exactly one point for all i , which is to say that f is constant on $[x_{i-1}, x_i]$ for all i . Finally, this implies that $f(a) = f(x_0) = f(x_1) = \dots = f(x_n) = f(b)$, which means that f is constant on the entire interval $[a, b]$.