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Mortality, Health, and Marriage: A Study Based on Taiwan's Population Data

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Life expectancy has been increasing significantly since the start of the 20th century, and mortality improvement trends are likely to continue in the 21st century. Stochastic mortality models are used frequently to predict the expansion in life expectancy. In addition to gender, age, period, and cohort are the three main risk factors considered in constructing mortality models. Other than these factors, it is also believed that marital status is related to health and longevity, and many studies have found that married persons have a lower mortality rate than the unmarried. In this study, we have used Taiwan's marital data for the whole population (married, unmarried, divorced/widowed) to evaluate if the marital status can be a preferred criteria. Furthermore, we also want to know whether the preferred criteria will be valid in the future. We chose two popular mortality models, the Lee-Carter and age-period-cohort, to model the mortality improvements for various marital statuses. Because of a linear dependence in the parameters of the age-period-cohort model, we used a computer simulation to choose the appropriate estimation method. Based on Taiwan's marital data, we found that married persons have significantly lower mortality rates than the single, and if converting the difference into a life insurance policy, the discount amount is even larger than that for smokers/nonsmokers.

1. INTRODUCTION

Prolongation of life expectancy has been a common phenomenon in many countries since the turn of the 20th century. For example, the life expectancies of U.S. males and females were both less than 50 years in 1900, whereas they reached about 74 and 80 years in 2000, respectively. On an average, males and females gained about 30 years of life over the past 100 years (Fig. 1), which is equivalent to gaining about 0.3 years of life annually. Life expectancies in the United States have been steadily growing and do not show apparent signs of slowing. The situation in Taiwan has been similar since the end of World War II. Life expectancies of males and females in Taiwan have also gained about 0.2 to 0.3 years annually (Fig. 1).

Traditional life insurance products are generally based on fixed mortality rates, and prolonged life expectancies create problems in calculating insurance premiums. For example, if someone purchases a deferred life annuity today at an age of 30 with benefits starting at the age of 60, there will be at least six years (assuming that the annual increment of life expectancy at age 60 is 0.2 years; $30 \times 0.2 = 6$) of difference in life expectancy if today's mortality rates are used to compute the insurance premiums. In other words, a risk exists of underestimating insurance premiums for life annuity policies and the insurers may go into insolvency, that is, longevity risk. Many previous studies suggested that mortality risk may cause substantial losses if handled improperly. See, for example, Huang et al. (2008) for a detailed discussion.

In recent years, intensive discussions have taken place around dealing with longevity risk. Using the dynamic life table, or, equivalently, the cohort life table, to replace the traditional life table with fixed mortality rates is one way of handling longevity risk. The search for a reliable mortality model is crucial for implementing the dynamic life table. Currently several mortality models are used, and the Lee-Carter (LC) model (Lee and Carter 1992) probably is the most popular choice.

If $m_{x,t}$ denotes the central death rate for a person age x at time t, then the LC model assumes that

$$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \varepsilon_{x,t}, \tag{1}$$

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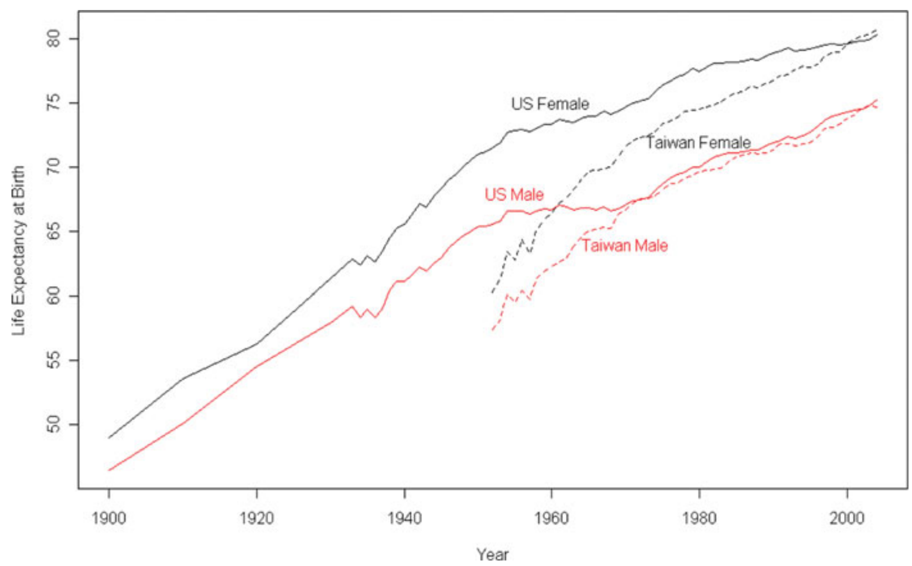

FIGURE 1. Life Expectancies in Taiwan and the United States.

where parameter α_x denotes the average age-specific mortality, κ_t represents the general mortality level, and the decline in mortality at age x is captured by β_x. The mortality level κ_t is usually a linear function in time. The term $\varepsilon_{x,t}$ denotes the deviation of the model from the observed log-central death rates and is assumed to be a white noise with 0 mean and a relatively small variance (Lee 2000). The LC model can be treated as a model with age effect, plus a mixed effect of both age and time.

Several modifications to the LC model have been made since it was first introduced in 1992. Many modifications are considered to include extra effects, in addition to the age and age-time effects. For example, Cairns et al. (2009) evaluated seven modifications (denoted M2 to M8) of the original LC model, using data from England and Wales and the United States. Among these modifications, two models (M2 and M8) that include a "cohort" effect have the best performances. However, unlike the LC model, where singular value decomposition was suggested, Cairns et al. did not give suggestions for the estimation of the model parameters.

Adding the cohort effect to the LC model, we find three effects: age, period, and cohort. This is similar to the age-period-cohort (APC) model in epidemiology. It is to be noted that these three effects would produce a linearly dependent structure, that is, age = time − cohort. It was shown that the estimation process was critical for giving reliable parameter estimates in the APC model. In fact, we found that the estimation process was also crucial in applying the modified LC models (such as M2 and M8).

In addition to age, period, and cohort, other factors are believed to be related to human longevity. For example, smokers generally have higher mortality rates and are charged with higher premiums for life policies than nonsmokers. Marital status is another risk factor, and many studies show that married persons have lower mortality rates (Gove 1973; Hu and Goldman 1990; Bennett et al. 1994; Trowbridge 1994; Cheung 2000; Gardner and Oswald 2004; Martikainen et al. 2005; Kaplan and Kronick 2006; Murphy et al. 2007). However, past marriage studies used sample data, and doubts have been expressed on sampling errors. In this study, Taiwan's marital and population data have been used to evaluate if marital status is a suitable choice for a preferred criteria.

Besides checking if the mortality rates are same for different marital statuses, we wanted to know if this preferred criteria will be valid in the future. We chose two popular mortality models, LC and APC models, to model the mortality improvements of various marital statuses. Because of the linear dependency of the parameters in the APC model, we used a computer simulation to choose an appropriate estimation method. Specifically, we examined if the estimation methods produced reliable and unbiased estimates for the model parameters.

2. THE APC MODEL

The APC model is used in epidemiology as a preliminary tool in disease incidence and mortality. It provides a means of descriptive statistics for summarizing information in a two-way table classified by age group and time period. In general, the APC model includes the three effects age, period, and cohort:

$$Y_{ij} = \log\left(R_{ij}\right) = \log\left(\frac{O_{ij}}{N_{ij}}\right) = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon_{ij}, \tag{2}$$

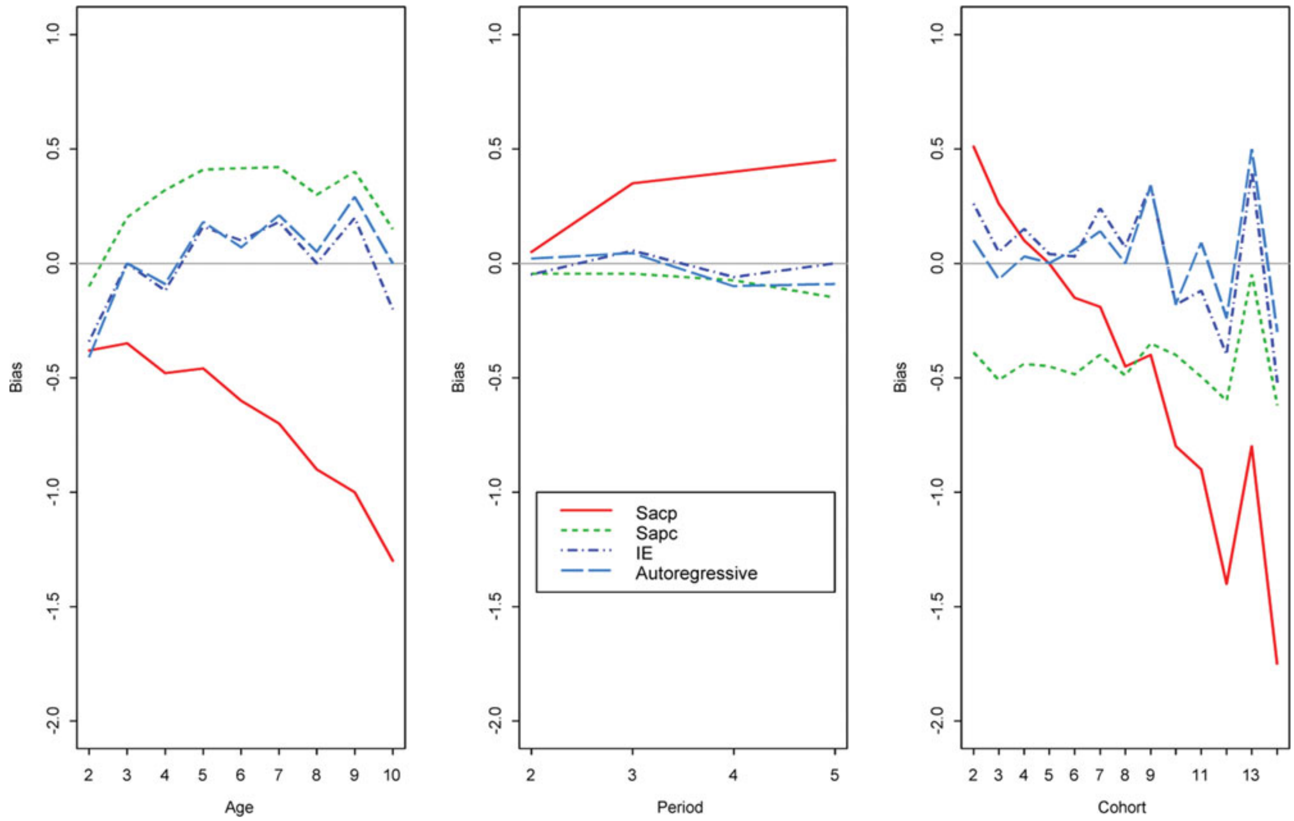

FIGURE 2. Bias of the Parameter Estimates for the APC Model. *Note:* Sacp and Sapc Are Sequential Approaches, with a, p, and c Indicating Age, Period, and Cohort.

where Y_{ij} is the logarithm of the death probability and $R_{ij}$is the death probability. Also, O_{ij} is the number of deaths for the *i*th age group ($i = 1, \ldots, m$), in the *j*th year ($j = 1, \ldots, n$), among N_{ij} people at risk, and it is generally assumed that O_{ij} follows a Poisson distribution. The model parameters, α_i, β_j, and γ_k each specify the effects of age, time (period), and cohort, respectively; ε_{ij} denotes a random error with expectation $E(\varepsilon_{ij}) = 0$ and $Var(\varepsilon_{ij}) = \sigma^2$. Note that the cohort effect γ_k satisfies $k = j - i + m$ and $k = 1, 2, \ldots, m + n - 1$. To avoid the problem in computing, it is usually assumed that $\sum_{i=1}^{m} \alpha_i = \sum_{j=1}^{n} \beta_j = \sum_{k=1}^{m+n-1} \gamma_k = 0$.

Since the three effects create a linear dependency, that is, determining two effects would automatically lead to determination of the third effect, an identification problem exists. Several methods have been proposed to solve the identification problem, such as the sequential method, autoregressive model, and individual record approach. Robertson et al. (1999) gave a thorough literature review of these methods, and they found that all these methods can give acceptable estimates of the parameters for nonlinear component. However, Robertson et al. did not consider the variances of the parameter estimates.

Yang et al. (2004) introduced a new method for solving the identification problem. The method is called the intrinsic estimator (IE), and it uses a generalized matrix inverse to acquire a unique solution of the parameters. Fu and Rohan (2004) showed the consistency and asymptotic properties of the IE and used simulation and empirical data to demonstrate that IE can yield satisfactory results. The estimation of the parameters and their variances can be achieved via principal component regression.

Although IE has been proven to have good statistical properties, no reported studies yet have compared IE to other solutions of the identification problem. In the next section, IE has been compared to two other solutions (sequential method and autoregressive model) through computer simulation. The simulation results can be used as a guideline in deriving the estimation method of adding the cohort effect to the LC model, as with the LC-cohort model proposed by Renshaw and Haberman (2005).

It is our belief that the APC model is a feasible and reliable model to describe the mortality trends of various marital statuses. This is one of the reasons to explore which method can provide satisfactory estimation results via a computer simulation in Section 3. To make sure the simulation settings match to (or are similar to) the Taiwan data, we first use the IE by Yang et al. (2004) to derive the estimates of parameters in the APC model (including the variance of errors). We then apply the appropriate estimation method to the Taiwan data in Section 4.

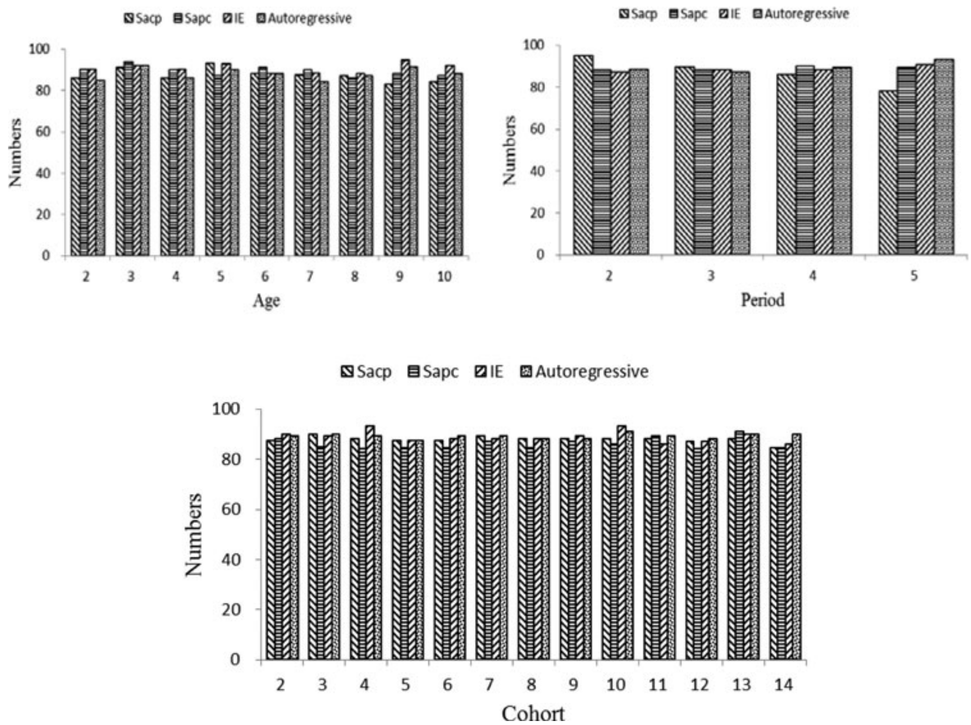

FIGURE 3. Numbers of Coverage for the APC Model Parameters (100 Runs, 90%).

3. SIMULATION

We chose two solutions for the identification problem, the sequential method and the autoregressive model, to compare with the IE The fitting processes for estimation methods are included in the Appendix. The process of applying the sequential method was similar to that of the approximation method (Lee and Carter 1992) for the LC model. The estimation of the sequential method can be done in the order of age-period-cohort (apc) or age-cohort-period (acp), according to the importance of the effects. This kind of estimation can be applied to model M2 in Cairns et al. (2009):

$$\ln(m_{x,t}) = a_x + b_x\ \kappa_t + \lambda_x \delta_{t-x} + \varepsilon_{x,t}, \tag{3}$$

where $m_{x,t}$ denotes the central death rate for a person aged x at time t, the parameters a_x,b_x, and $\lambda_x$denote the average age-specific mortality, κ_t represents the general mortality level, and δ_{t-x} reflects cohort-related effects; the age, period, and cohort effects (i.e., a_x, κ_t, δ_{t-x}) can be estimated using the sequential method.

The period effect, κ_t, in the LC model is usually a linear function of time and is an autoregressive effect. Similarly, the autoregressive model of the APC model assumes that γ_k satisfies $\gamma_k = \varphi\gamma_{k-1} + e_k$, with $e_k \overset{i.i.d}{\sim} N(0, \sigma^2)$, still keeping the constraints $\sum_{i=1}^{m} \alpha_i = \sum_{j=1}^{n} \beta_j = 0$. The autoregressive relationship was applied to the cohort effect because the adjacent cohorts are likely to share similar properties and the correlation between these cohorts should be positive, or $\varphi > 0$.

There are 10 five-age groups, five periods, and 14 cohorts in the simulation. The age effect is a U-shaped curve, like the usual mortality curve, the period effect is a (decreasing) linear function of time, and the cohort effect looks like an upside-down U-shaped curve. These parameter settings mimic the three effects in Taiwan. The simulation was repeated 100 times, assuming that the variance in (2) satisfies $\varepsilon_{ij} \sim N(0, 3)$. The sample variance fitting the Taiwan data was close to 3, and thus we assumed that the errors followed the normal distribution $N(0,3)$. Note that computations and simulations in this study were run on a Windows PC.

The bias and coverage probability were checked to evaluate the sequential method, autoregressive model, and IE. Figure 2 shows the average bias from 100 simulation runs. On average, the autoregressive model and IE displayed the best performances, and the age, period, and cohort effects were close to unbiased. The Sapc sequential method gave satisfactory results, but the age effects were slightly overbiased and the cohort effects were underbiased. The Sacp sequential method was not as good, with the age and cohort effects underbiased and the period effects overbiased. The results of the sequential method were similar to those in the M2 model, where the order of apc would create more satisfactory results than acp.

TABLE 1
Taiwan's Mortality Data by Marital Status

Year	Population	Deaths
1973–1974	15–50+	N/A
1975–1991	15–50+	15–85+
1992–1993	15–50+	15–95+
1994–1997	15–100+	15–95+
1998–2006	15–100+	15–100+

To see if the estimation methods produced acceptable results, we also used the coverage probability as a check (Fig. 3). The confidence coefficient was set to 90%. Generally, for 100 simulation runs, the estimation results are satisfactory if the numbers of coverage are between 84 and 96. In some occasions of age, period, and cohort effects, the Sacp sequential method showed numbers of coverage smaller than expected. The other three methods gave satisfactory results, and the autoregressive model and IE had numbers of coverage close to the nominal value. We recommend using these two methods, and only IE will be used in the rest of this study.

4. EMPIRICAL STUDY

In this section, Taiwan's data have been used to explore the mortality rates of marital statuses, followed by application of the APC model to evaluate their mortality trend. Furthermore, only the IE method has been used for the marital data because the IE and autoregressive methods produce almost identical estimates. It was recognized that mortality rates are highly correlated to the marital status. According to Trowbridge (1994), the possible explanations for married persons having lower mortality include selection at marriage, responsibility, living arrangements and reciprocal care giving, mutual interactions, and social interactions. In recent years, the percentage of married males and females in Taiwan has been decreasing significantly. For example, for the age group 25–29, the percentage of married females dropped from 90% in the 1970s to 40% in the 2000s. Overall life expectancies, however, continue to increase every year. It would be interesting to see if married persons still have lower mortality rates than those of other marital statuses.

The mortality data by marital status in this study are from the Ministry of the Interior, Government of Taiwan. There are four marital statuses: single, married, divorced, and widowed. Considering the data size, we combined divorced and widowed into one

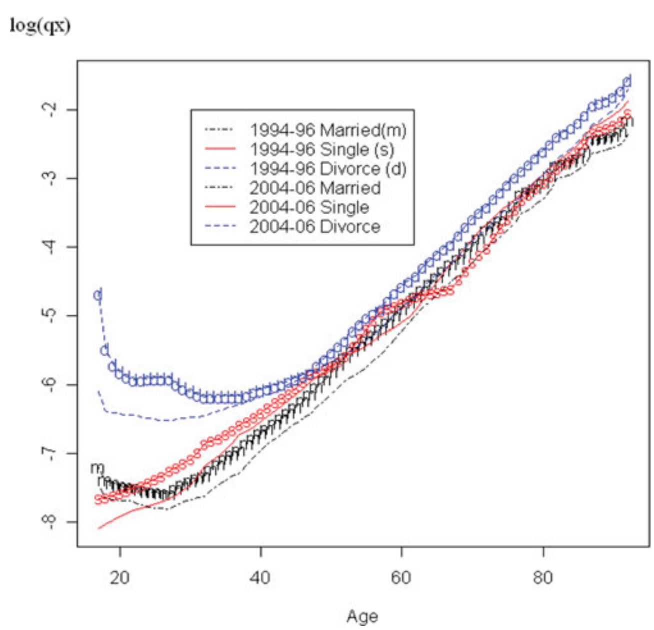

FIGURE 4. Female Mortality Rates by Marital Status in 1994–1996 and 2004–2006.

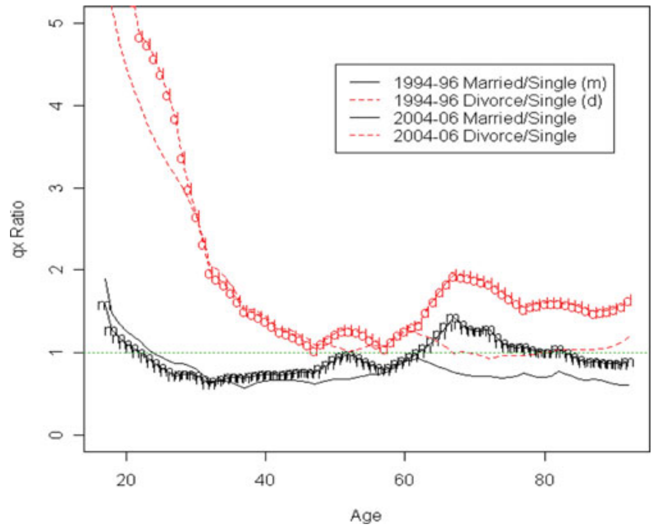

FIGURE 5. Female Mortality Ratio in 1994–1996 and 2004–2006 by Marital Status.

group, divorced/widowed. Table 1 lists the records of populations and deaths since 1973, the first year the data are available by marriage status.

Since detailed mortality records are available from 1994 onward, the mortality rates between the periods 1994–1996 and 2004–2006 were first compared. The results of females were used as a demonstration (Fig. 4). The mortality rates by marital status in 1994–1996 were marked with letters (m: married, s: single, d: divorced/widowed). The groups of married and divorced/widowed persons had lower mortality rates in all age groups from 1994–1996 to 2004–2006. The single group had lower mortality rates for ages lower than 50 but have obviously higher rates for females aged 60 to 80. It seemed that mortality improvement did not occur for the groups of ages before 50 and older ages 65–80 for single females in Taiwan.

To further investigate the differences in mortality improvements by marital status over 1994–1996 and 2004–2006, we computed the mortality ratio (Fig. 5). The mortality rates of the single were treated as a standard group. The ratios of married versus single and divorced/widowed versus single declined from 1994–1996 to 2004–2006, except for younger age groups (ages less than 30). A further examination of the mortality rates for married versus single persons over a longer period (1975–2006; Fig. 6) shows similar results; married males have larger mortality improvements as well. This indicated that the single group in general had the least mortality improvement from 1994–1996 to 2004–2006. Also, comparing with the single group, the mortality reduction

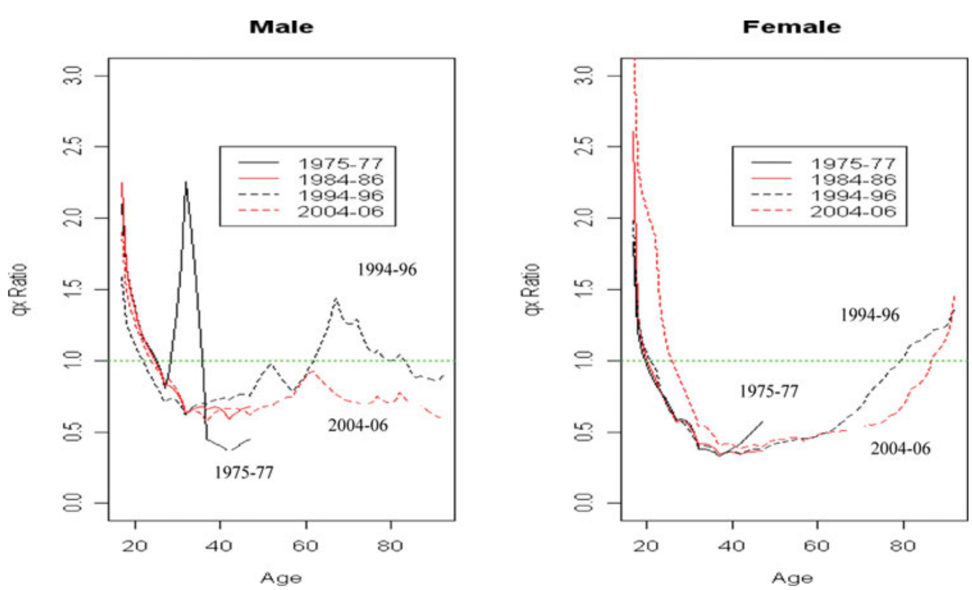

FIGURE 6. Male and Female Mortality Ratios (Married vs. Single).

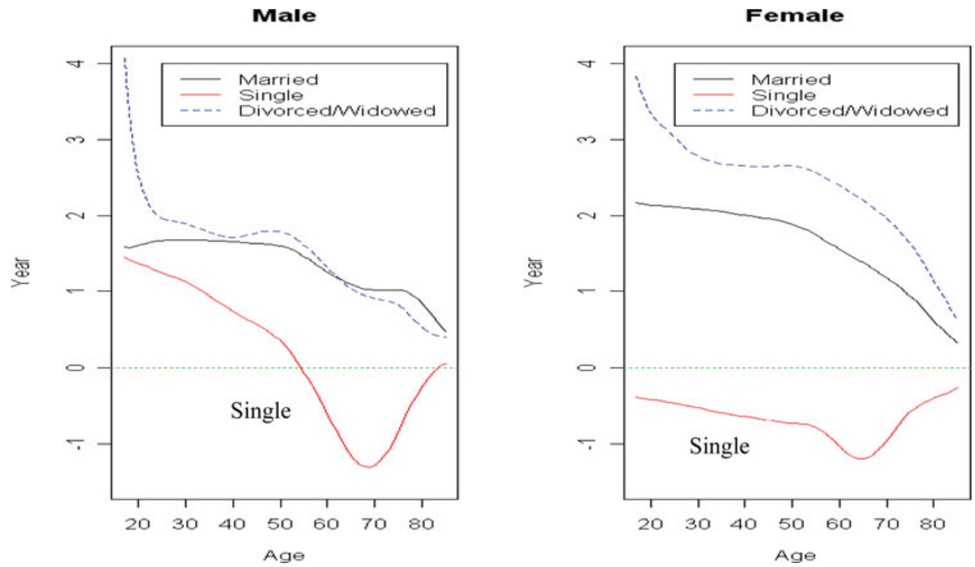

FIGURE 7. Increase in Life Expectancy by Marital Status (1994–1996 to 2004–2006).

was especially noticeable at ages over 40 for the married group and at ages over 60 for the divorced/widowed group. In the past, divorced/widowed persons were believed to have the largest mortality rates, but they had almost the same mortality rates as that of singles for ages 50 and beyond in 2004–2006.

Figure 7 shows the gain in life expectancies by marital status over the 10-year period (1994–1996 to 2004–2006). Divorced/widowed females have the largest increase in life expectancies, and divorced/widowed males, married males, and married females all showed a similar large increase. Single males still had positive gains for the younger age groups, but single females experienced decreases in life expectancy for all ages. This indicated that the mortality improvements were not homogeneous by marital status. Both married and divorced/widowed persons showed noticeable mortality improvements from 1994–1996 to 2004–2006.

In addition to the comparison of mortality rates, we also used the APC model to measure the period and cohort effects of the mortality improvement by marital status. As shown in the previous section, IE was chosen as the estimation method since it displayed better performances. Additionally, it requires a certain amount of observations to apply the APC model, and thus the data in the format of five-age groups (ages 15–50+, eight groups) and five-year periods (year 1975–2005, seven data points) were chosen to guarantee a sufficient data size.

To see the annual change in age-specific mortality rates, only the sum of period and cohort effects was computed, because, according to Equation (2), only the period and cohort effects are time dependent. The exponential value of this sum is equal to the reduction rate, or $1 - \frac{m_{x,t+1}}{m_{x,t}} = 1 - \exp(\Delta\beta_t + \Delta\gamma_{t-x})$. For example, if the sum is -0.07 for married persons, then, on average, they would have a reduction of $1 - e^{-0.07} \cong 6.76$ % in mortality rates for all ages every five years. Table 2 lists the averages of period and cohort effects over 1975–2005 for three marital groups. The divorced/widowed group had the largest reduction, which were 7.3% and 7.7% every five years for males and females, respectively. Married males and single males exhibited a very close reduction rate, 2.0% and 2.5%, respectively. However, married females showed a larger reduction rate than single females. These results were slightly different than those in Figure 4 (1994–1996 vs. 2004–2006) because more observations (1975–2005) were used and the estimates of "period + cohort" effects would be different.

TABLE 2
Estimation of Period and Cohort Effects by Marital Status

	Married	Single	Divorced/Widowed
Male	−0.0196	−0.0251	−0.0762
Female	−0.0697	−0.0510	−0.0798

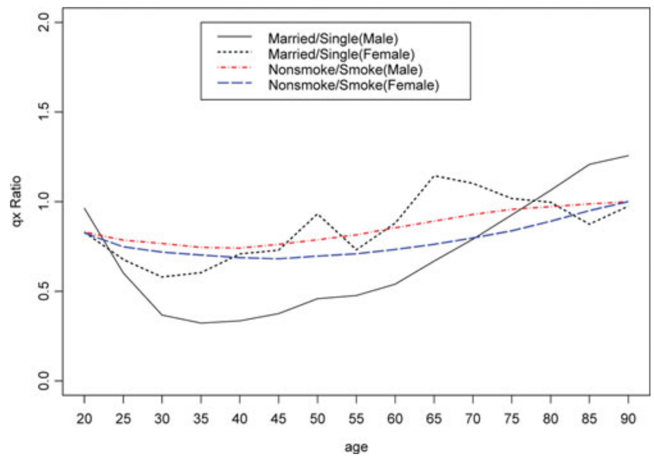

FIGURE 8. Mortality Ratios of Marital and Smoking Status.

5. APPLICATIONS OF MARRIAGE MORTALITY RATE

The aging population in Taiwan has increased the burden of social insurance in the country. For example, the government raised the premiums for the National Health Insurance and National Pension Insurance in early 2013, and it is likely that those premiums will be raised again in the near future. It seems that social insurance cannot (and should not) take care of all of the people's needs. Unfortunately, about one-third of people in Taiwan do not own any insurance policies, and those who have insurance policies are underinsured (Yue and Huang 2011). To reduce the burden of public finance, Taiwan's Insurance Bureau, Financial Supervisory Commission, in 2007 started to encourage life insurance companies to develop preferred-status life insurance products, hoping that the premium discount would raise demand for insurance products.

Two risk factors are usually considered in preferred-status insurance, smoking and being overweight (obesity). However, these two factors in Taiwan are rarely listed on the health exam form, and the responses to the question if someone smokes are often doubted by the insurers. At least 25% of insured do not honestly disclose if they currently smoke, according to some insurers

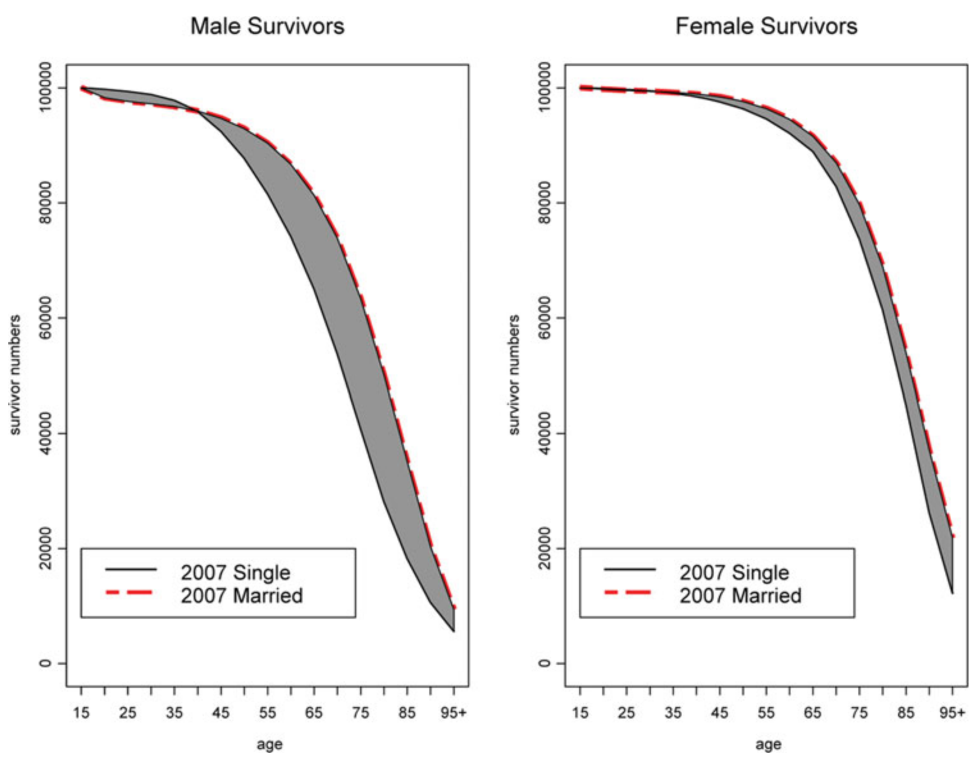

FIGURE 9. Number of Survivors for Marital Status.

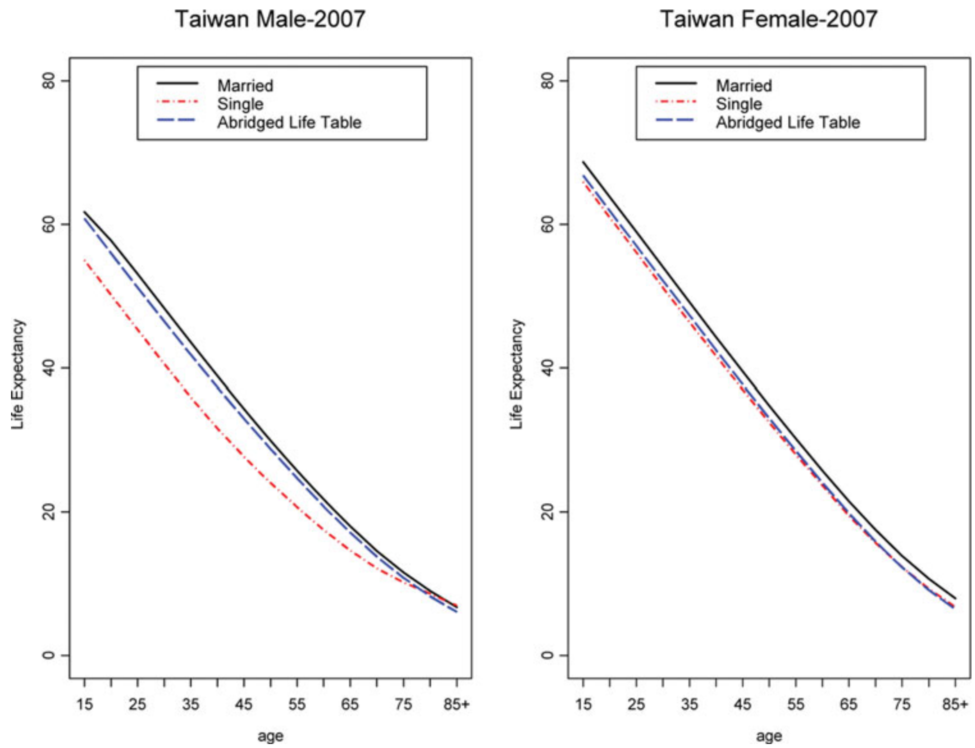

FIGURE 10. Life Expectancies of Various Marital Statuses.

(Schwaiger 2005). The information if a person is married is easier to collect (it can be found in the population registration system), and thus marital status can be used in pricing preferred-status insurance. As mentioned in the previous section, Taiwan's mortality data for marital status are a record of the whole population (i.e., census data) and do not suffer from the drawback of selection bias in most sample data. Of course, the mortality rates calculated in the last section require some adjustments before they can be used to price insurance products.

The influence of marriage has been studied for a long time, and generally it is believed that marriage has a role in health protection (Lillard and Panis 1996; Williams and Umberson 2004; Henretta 2007; Manzoli et al. 2007; Van den Berg and Gupta 2007; Liu and Umberson 2008) and a married person lives longer than an unmarried one or has a lower mortality rate (Gove 1973; Hu and Goldman 1990; Bennett et al. 1994; Trowbridge 1994; Cheung 2000; Gardner and Oswald 2004; Martikainen et al. 2005; Kaplan and Kronick 2006; Murphy et al. 2007). Therefore, it appears that using marital status as a risk factor in preferred-status insurance is a suitable choice.

TABLE 3
Premiums for 20-Year-Pay Whole Life (Interest Rate: 5%)

	Male				Female			
Age	Married	Single	Married/Single	Married/HMD	Married	Single	Married/Single	Married/HMD
30	23.13	37.06	0.62	0.84	15.52	19.17	0.81	0.92
35	28.78	48.19	0.60	0.86	19.40	23.54	0.82	0.93
40	35.48	61.12	0.58	0.88	24.24	28.64	0.85	0.94
45	43.83	75.70	0.58	0.90	30.21	35.00	0.86	0.94
50	55.18	93.68	0.59	0.91	37.90	43.52	0.87	0.94
55	71.31	116.99	0.61	0.93	48.57	55.10	0.88	0.93
60	94.11	151.31	0.62	0.93	64.01	74.34	0.86	0.92

TABLE 4
Premiums for 20-Year-Pay Whole Life (Interest Rate: 2%)

	Male				Female			
Age	Married	Single	Married/Single	Married/HMD	Married	Single	Married/Single	Married/HMD
30	37.28	49.36	0.76	0.91	30.60	33.81	0.91	0.96
35	42.14	59.25	0.71	0.91	34.10	37.67	0.91	0.96
40	47.89	70.99	0.67	0.91	38.28	42.05	0.91	0.96
45	55.23	84.65	0.65	0.92	43.35	47.43	0.91	0.96
50	65.53	102.04	0.64	0.93	49.96	54.84	0.91	0.96
55	80.76	125.34	0.64	0.94	59.48	65.56	0.91	0.95
60	103.03	159.90	0.64	0.93	73.93	84.03	0.88	0.93

In relation to the above context, we first calculated the differences of age-specific mortality rates between married and single persons and then compared them with those between nonsmoking and smoking persons. Note that the mortality data of smoker status were based on the claim experience of Taiwan insurance companies. We used the mortality ratio to check the differences in mortality rates for different marital and smoking statuses (Fig. 8). In general, married persons have smaller mortality rates than single ones, and this difference in mortality rates was about the same between nonsmokers and smokers. Males apparently benefited more from marriage, and the mortality rates of married males were about 40% of those for the single male around ages 30–50. The results showed that marital status is a feasible choice for designing the preferred-status insurance, especially for those between 30 and 60 years old.

The impact of using marital status in designing life insurance products can be evaluated by the survival curves. Figure 9 shows the survival curves, starting from age 15, of single and married persons in 2007. The areas between the survival curves (gray shading) can be treated as the differences in life expectancy between single and married persons, and they matched with the results of life expectancy in Figure 10. The area between married and single persons was larger for males, similar to that in Figure 8. The advantage of marriage started at around age 40, and it quickly accumulated. We expect that the premiums of life insurance products for married persons would be significantly lower than those of single persons.

We used whole life insurance to demonstrate the premium differences of using marital status for preferred-status insurance. The married and single life tables were constructed using the Whittaker graduation, where the exposures of raw mortality rates were based on the assumption of 30/360 or 30 days per month and 360 days per year. The graduation process involved calculation of the raw mortality rates of five-age groups first. After graduating five-age mortality rates, we used the interpolation formula (cubic spline) and the assumption uniform death distribution to obtain age-specific mortality rates for a single age. Note that the mortality rates of people aged 80 and older were graduated using the Gompertz law assumption. The estimation of parameters using the Gompertz law was done with weighted least squares (Yue 2002).

The insured ages for whole life insurance considered were from 30 to 60, with a payment period of 20 years, and the interest rate was 2% or 5%. Tables 3 and 4 are the annual pure premiums for coverage amount per $1,000. As expected, married males had a larger discount in their premiums, and the discount was about 40% and 35% for 5% and 2% interest rates, respectively. To avoid any marriage discrimination, we also compared married persons with respect to the whole population, with population data from the Human Mortality Database (HMD). In other words, this would treat all insureds equally and give discounts only to married persons. Given a 5% interest rate, the premium discounts for married persons versus the whole population were about 7–16% and 6–8% for males and females, respectively. The discounts for the case of 2% interest were smaller.

6. DISCUSSIONS AND CONCLUSIONS

Mortality improvement is a common phenomenon in many countries, and people are expected to live longer. However, this prolonged life expectancy puts insurers into a high risk of insolvency if the longevity risk is underestimated. Because of rapid population aging and the growing need for life annuity products, the insurance industry needs to figure out solutions to minimize this risk. The dynamic life table (or cohort life table) is one possible solution, and most of these tables rely on mortality models. The Lee-Carter (LC) model is probably the most popular mortality model, and several modifications to the LC model have been proposed. Among these modifications, introduction of the “cohort” effect into the model is a common variation of the LC model, in addition to age and period effects. However, since the three effects create a linear dependency, a careful estimation of these effects is necessary.

The age-period-cohort (APC) model is a frequently used tool in epidemiology, and many solutions have been proposed to deal with the above-mentioned linear dependency. In this study, we used computer simulations to evaluate the estimation methods of the APC model, including a recently proposed method, the intrinsic estimator (IE). Judging from the bias and coverage probability, we found that IE is a feasible method, and the estimates and their variances can be derived directly. The autoregressive model can be treated as an alternative method for the parameter estimation. We suggested the use of IE and autoregressive models to deal with the problem of linear dependency after the cohort effect was introduced in the LC model.

The empirical study of Taiwan mortality by marriage status suggested that the divorced/widowed had the largest gains in life expectancy. By contrast, married persons' mortality rates at all age groups declined significantly. On the other hand, the single had the least mortality improvement among all the three marital statuses, and the single elderly even experienced an increase in mortality rates. Applying the mortality data by marital status to the APC model, the obtained results were similar, and the divorced/widowed had the largest mortality reductions, but the differences between married males and single males were smaller. The results of applying the LC model were similar and have been omitted here.

Married males would receive a 24% to 42% discount in pure premium above single males. It seemed that married or not can be regarded as a mortality risk factor for males. A positive side of this research is the use of the complete population record data in Taiwan to analyze the relationship of marriage status and mortality. We have the same results as have had scholars in many countries. Hence, we suggest that future social security disbursement should also take marital status into consideration and provide insurance companies with references in designing life insurance products. It would also indirectly encourage motivation to get married to increase the fertility rate.

Furthermore, we used a computer simulation to evaluate the estimation methods of the APC model. The goal was to study the estimation methods and provide possible suggestions for the LC model, if the cohort effect is to be introduced into the mortality model. We found that the IE and autoregressive models outperformed the sequential approach in parameter estimation of the APC model. Since adding the cohort effect in the LC model usually adapts an approach similar to the sequential APC model (Renshaw and Haberman 2005; Cairns et al. 2009), there may be room for improving the estimation. Of course, we do not suggest that the APC model is a better model or adding the cohort effect is the only way to modify the LC model. For example, Debón et al. (2008) proposed a geostatistical modification to the LC model, adding a spatial autocorrelation to the residuals of the LC model.

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APPENDIX: FITTING PROCESSES FOR ESTIMATION METHODS OF THE APC MODEL

(a) The Sequential Method: Because of the linear dependency, we can first estimate two effects (either AP or AC) and then estimate the remaining third effect. We will call these two approaches the sequential APC (Sapc) and sequential ACP (Sacp) approach. Carstensen and Keiding (2005) suggested using the generalized linear model to fit the Sacp approach, through the following steps:

Step 1. Choose a reference cohort k_0 and fit the AC model. Then we have the estimates of period and cohort parameters, $\hat{\alpha}_i$ and $\hat{\gamma}_k$.
Step 2. Set $\hat{\alpha}_i + \hat{\gamma}_k + \log(N_{ij})$ as an offset to fit model, and we have the period parameters estimation $\hat{\beta}_j$.

(b) Autoregressive Model: We refer to Lee and Lin (1996, pp. 257–258) for adopting the conditional likelihood approach for estimation of the parameters. The conditional log-likelihood function of the proposed model Equation (2) is as follows:

$$\log L(\mu, \alpha, \beta, \gamma, \phi, \sigma^2/y) = -\frac{m+n-1}{2}\log 2\pi + \frac{1}{2}\log(1-\phi^2) - \frac{m+n-1}{2}\log\sigma^2 - \frac{S}{2\sigma^2} + \sum_{i=1}^{m}\sum_{j=1}^{n}(y_{ij}\log O_{ij} - O_{ij} - \log y_{ij}!),$$

where $S = \gamma_1^2(1-\phi^2) + \sum_{k=2}^{m+n-1}(\gamma_k - \phi\gamma_{k-1})^2$ and y_{ij} follows Poisson distribution with mean $O_{ij} = N_{ij}R_{ij}$. We then use the Levenberg-Marquardt algorithm (Thisted 1988) for maximization, which ensures that we always move in an ascending direction on the likelihood surface. We may use, for example, the Newton-Raphson method to obtain the parameter estimates.

(c) Intrinsic Estimator: Using the vector space projection, this approach yields a unique solution to model Equation (2) that is determined by the Moore-Penrose generalized inverse and removes the arbitrariness of linear constraints from the parameters. The intrinsic estimates of model regression coefficients and their standard errors are computed via the principal components regression algorithm introduced by Yang et al. (2004).

Equation (2) can be written in the conventional matrix form of a least-squares regression: $Y = Xb + \varepsilon$, where Y is a vector of mortality rates or log-transformed rates, X is the regression design matrix consisting of "dummy variable" column vectors for the vector of model parameters b:$b = (\mu, \alpha_1, \ldots \alpha_{m-1}, \beta_1, \ldots, \beta_{n-1}, \gamma_1, \ldots, \gamma_{m+n-2})^T$, with the superscript T denoting vector transposition, and ε is a vector of random errors with mean 0 and constant diagonal variance matrix $\sigma^2 I$, I being the identity matrix. The IE equals $\hat{b} = B + tB_0$. The special intrinsic estimator B is determined by the Moore-Penrose generalized inverse and t is an arbitrary real number; B_0 is the eigenvector corresponding to the zero eigenvalue of X.

The computational algorithm can be seen in Appendix B of Yang (2006, p. 35). The steps are as follows:

Step 1. Compute the eigenvectors $u_1, \ldots, u_r$ of matrix $X^T X$. Normalize them and denote the orthonormal matrix as $U = (u_1, \ldots u_r)^T$.

Step 2. Identify the special eigenvector B_0 corresponding to eigenvalue 0. Let $u_1 = B_0$.

Step 3. Fit a principal components regression model using a design matrix V whose column vectors are the principal components $(u_2, \ldots, u_r)$, to obtain the coefficients$(w_2, \ldots, w_r)$.

Step 4. Set $w_1 = 0$ and transform the coefficients vector $w = (w_1, \ldots, w_r)^T$ by the orthonormal matrix $U = (u_1, \ldots u_r)^T$ to obtain the intrinsic estimator $B = Uw$.