Mortality Compression and Longevity Risk

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ABSTRACT

Mortality improvements, especially of the elderly, have been a common phenomenon since the end of World War II. The longevity risk becomes a major concern in many countries because of underestimating the scale and speed of prolonged life. In this study, we explore the increasing life expectancy by examining the basic properties of survival curves. Specifically, we check if there are signs of mortality compression (i.e., rectangularization of the survival curve) and evaluate what it means to designing annuity products. Based on the raw mortality rates, we propose an approach to verify if there is mortality compression. We then apply the proposed method to the mortality rates of Japan, Sweden and the United States (data source: Human Mortality Database). Unlike the previous results using the graduated mortality rates, we found there are no obvious signs that mortality improvements are slowing down. This indicates that human longevity is likely to increase and longevity risk should be seriously considered in pricing annuity products.

Key Words: Mortality Improvement; Longevity Risk; Mortality Compression; Graduation, Mortality Models

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1. Introduction

Human life expectancy has been experiencing a steady and probably the longest increase in history since the turn of the 20th century. It is believed many factors contribute to the prolonging of life, such as medical and environmental improvements. For example, the life expectancies of both U.S. males and females were less than 50 years at the beginning of the 1900s and have increased rapidly to be higher than 70 at the end of 20th century (Figure 1). The increments are about 30 years in 100 years, which is about 0.3 years annually. The trend is likely to continue, at least for a while, and the life expectancies of males and females at the end of 2008 are 75 and 81, respectively (CIA World Factbook).[2] The phenomenon of prolonging of life also appears in other countries. According to the United Nations, on average, life expectancy had an increment of 0.25 years annually during the 20th century.

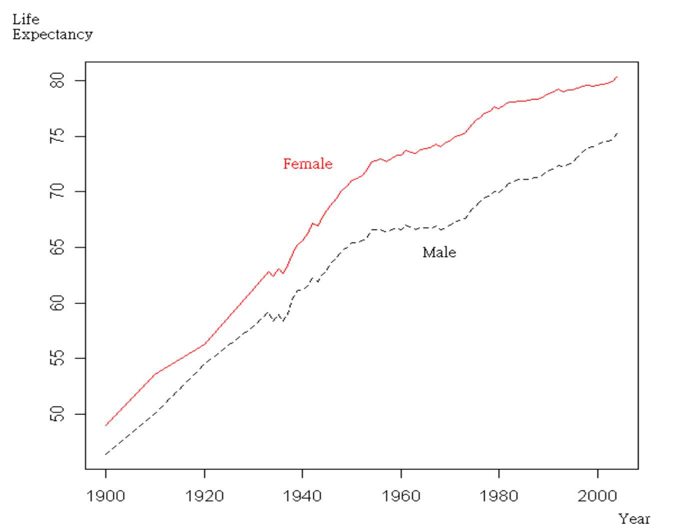


Figure 1. Life Expectancy in the U.S. in the 20th Century

The prolonging of life has dramatically changed individual's life planning, especially for retirement, in the 21st century. Many believed that the life limit is 85, and some social welfare related systems and annuity products were designed according to this conjecture.

2 Data Source: https://www.cia.gov/library/publications/the-world-factbook/fields/2102.html

Recently, many countries started to feel the pressures of underestimating the scale and speed of prolonged life. Japan, known as the longest living country now, had a much shorter lifespan and did not anticipate the rapidly decreasing mortality. As a result, some insurance companies face financial insolvency. The social security system in the U.S. faces a similar problem, partly due to the elderly experiencing lower mortality than expected. According to the 2012 annual report from the Social Security Board of Trustees, the combined Trust Funds are expected to be exhausted in 2033 – three years sooner than projected in 2011. If the trend of reducing mortality continues its pace, there will be more countries facing similar problems.

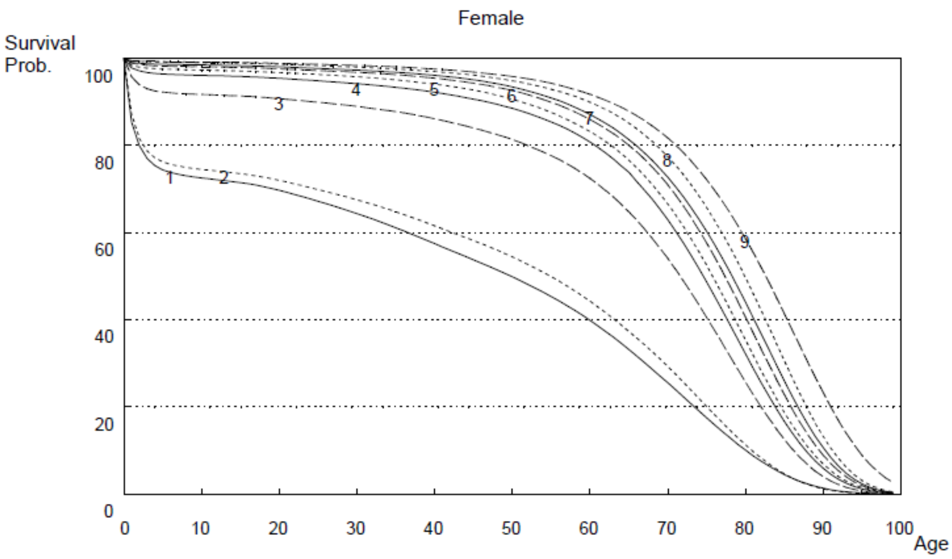


Figure 2. Survival Curves of Taiwanese Females

However, because of the rapid increase in life expectancy over such a short period, there is not enough data to model the mortality rates of the elderly. There are even less data available for the oldest-old (people 85 and older) for exploring if life expectancy has a limit. Therefore, there is still no consensus about the life limit. Nonetheless, researchers believe many countries do share some common features in mortality. Rectangularization of the survival curve, or mortality compression, is one of them. According to Fries (1980), this is a state in which mortality from exogenous causes is eliminated and the remaining variability in the age at death is caused by genetic factors. Take the survival curves in Taiwanese females as an example (Figure 2)[3]. It is obvious the mortality rates for infants and children have lessened significantly from years 1920 to 2000 (lines 1 to 9, with an increment of 10 years from numbers 1 to 9), and the survival probability for an infant to age 50 is more than 90 percent in 2000. The majority of deaths occur between ages 65 and 85, accounting for about 50 percent of deaths in 2000.

In addition to describing the significant mortality reduction in premature ages, some researchers use rectangularization to measure if human life has a limit. Kannisto (2000) proposed some statistics to check if human life has a limit. He found that, although life expectancies of the U.S. and European Union countries continue to increase, more deaths occur in a shorter age interval (i.e., mortality compression). Cheung et al. (2005) proposed three-dimensional measures for the survival curve, including horizontalization, verticalization and longevity extension. They applied the data from Hong Kong and also confirmed mortality compression. However, the number of survivors l_x and number of deaths d_x used in the past studies are all from life tables, not the raw data. The numbers found in life tables are all graduated values and may vary a lot if choosing different graduation methods.

The graduation would have a larger impact on the mortality of the elderly, since there are fewer observations available. The graduated mortality rates may differ greatly, if different graduation methods or criteria are used. We shall use the 1999-2001 Complete Life Table for Taiwanese males as a demonstration. Figure 3 shows the mortality rates of ages 70 to 99, using different graduation methods. The mortality rates show obvious differences starting at age 80 and can differ almost up to 100 percent around age 99. As a result, the numbers of survivors and deaths converted from the mortality rates also have many

[3] The Taiwanese complete life tables, including the survival curves and related statistics in this study, are from the Ministry of Interior, Taiwanese Government. (http://sowf.moi.gov.tw/stat/Life/List.html; data retrieved on June 15, 2012.)

differences and thus influence the measures of rectangularization. Note that, in addition to the graduation methods, the assumption of ultimate age in constructing the life tables can also affect the numbers of survivors and deaths.

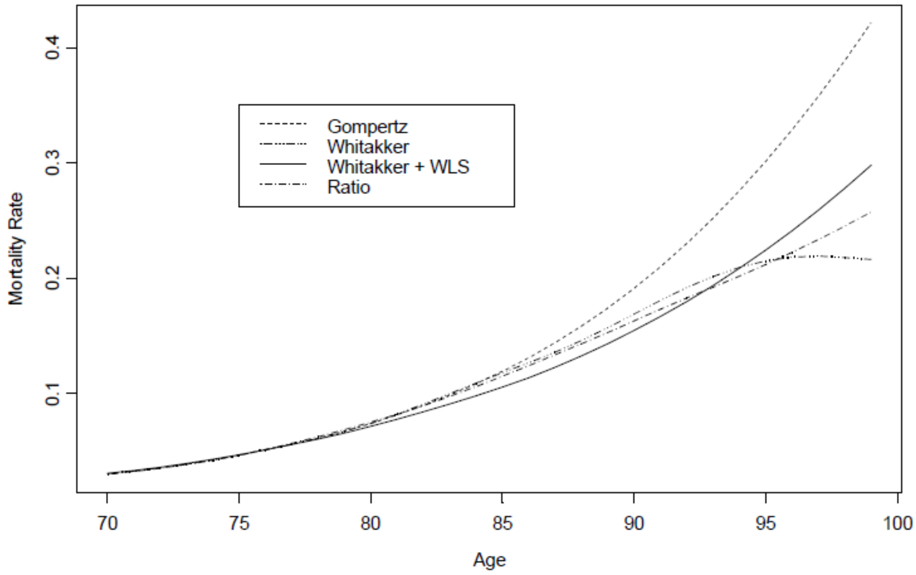


Figure 3. Mortality Rates of Taiwanese Males (1999-2001 Complete Life Table)

In this paper, to avoid the influence of graduation methods, we propose using the raw data from several countries to measure rectangularization. For the rest of this manuscript, we shall first introduce the past work related to mortality compression and potential limitations of the methods used. We will describe the proposed measurements for evaluating rectangularization and show the results of empirical analysis in Section 3. We then discuss the implications of our findings on insurance in Section 4, followed by discussions and limitations of the study in Section 5.

2. Review of Mortality Compression Studies

The rectangularization of the survival curve indicates death ages are more concentrated and are with less variability (or variance). This idea was first proposed by Fries (1980). He believed that by eliminating exogenous causes of death, everyone should have about the same life expectancy, excluding the variability from genetic factors. He also thought life expectancies were close to a limit, based on data from the U.S. and Europe. In addition, he hypothesized lifetime illness will be compressed into a shorter period before death (morbidity compression). Although some studies claimed they have proved Fries' hypotheses (Vita et al., 1998), the issues of mortality and morbidity compressions remain controversial and there are no concrete evidences.

To study the compressions of mortality and morbidity, Wilmoth and Horiuchi (1999) proposed 10 measurements on the survival curves to evaluate whether the variability of deaths and illness is less. The measurements include six in quantifying rectangularity and four in computing variability. Wilmoth and Horiuchi suggested using the interquartile (IQR), or the difference of first and third quartiles, to measure if there are compressions.

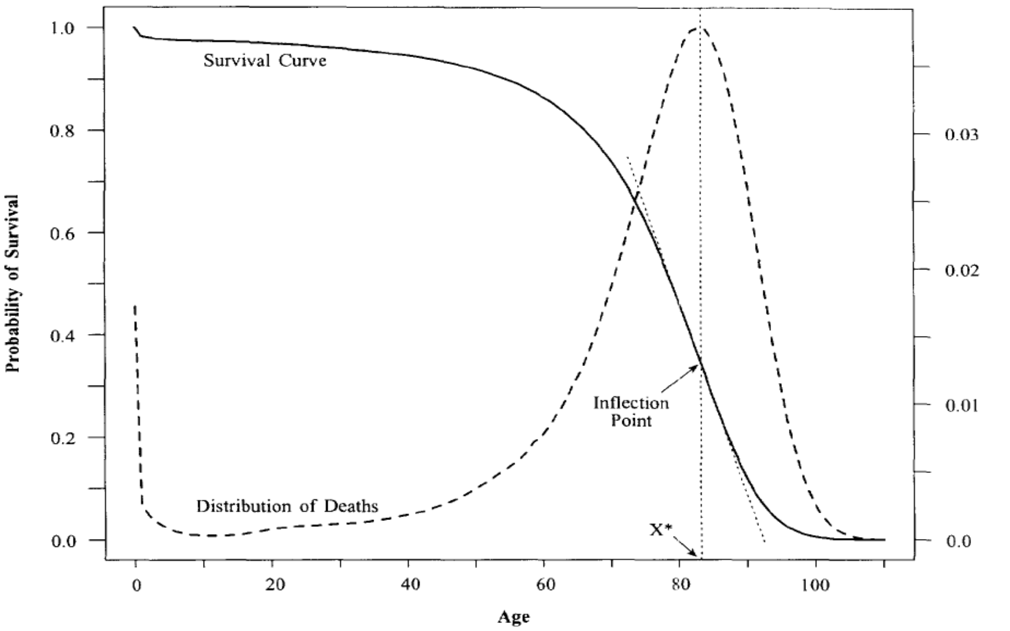


Figure 4. Survival and Mortality Curves (Source: Wilmoth and Horiuchi, 1999)

Kannisto (2000, 2001) applied the numbers of deaths from life tables and calculated the IQR and varies percentiles. He confirmed the mortality compression, using the data from 22 countries. For example, Figure 5 shows the numbers of deaths for Swedish males, from the

1861-70 and 1991-95 life tables. It is obvious that both the IQR and the smaller number of ages covering 50 percent of deaths, denoted as C_{50}, are smaller in years 1991-95. This indicates that the ages of deaths are becoming more condensed. Kannisto also concluded that the numbers of deaths on the right side of mode (age with highest values) do resemble the shape of a normal curve.

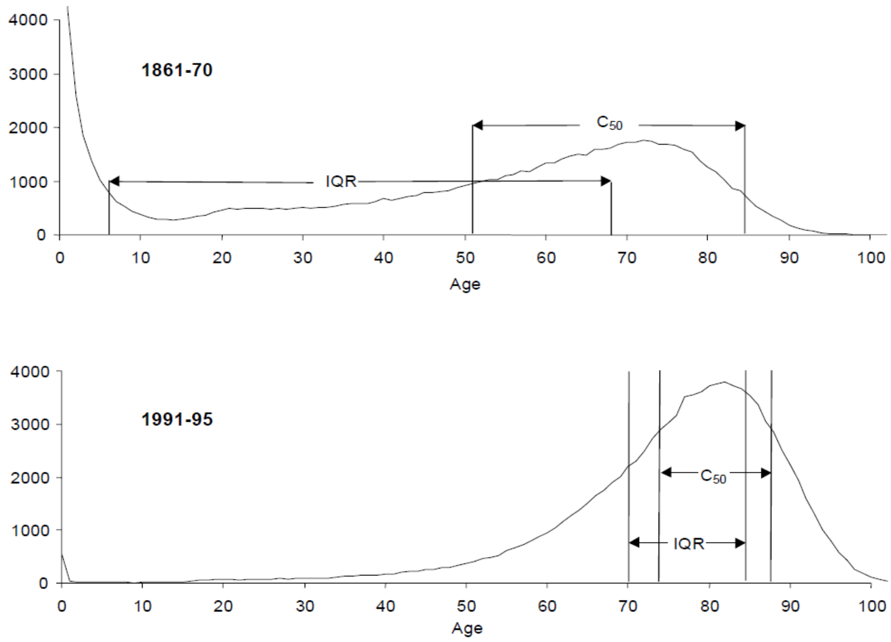


Figure 5. Numbers of Deaths of Swedish Males (Source: Kannisto, 2000)

Cheung et al. (2005) modified the ideas from the previous work and proposed three-dimensional measurements for the change in survival curves: horizontalization, verticalization and longevity extension (Figure 6). The horizontalization is to measure the proportion of premature deaths, such as the percentage of deaths before age 50. It can be defined as the angle between the horizontal line and survival curve (β in Figure 6), and a smaller β indicates fewer premature deaths. The verticalization is the age with the maximum number of deaths, i.e., mode, or is to measure the descending speed at the mode (θ and θ^* in Figure 6). A smaller value of θ and θ^* indicates there are more deaths occurring around the modal age (i.e., the mortality compression).

Cheung et al. used the complete life tables from 1976-2001 in Hong Kong and found the verticalization is more obvious than the horizontalization, indicating the longevity extension seems to slow down. Note that, since there is insufficient data, the mortality rates of ages 85 to 120 in Hong Kong are graduated using the logistic curve. Note that *M+* in Figure 6 corresponds to changes in the right-hand tail of the survival curve and describes how far the highest normal life durations can exceed the modal age at death.

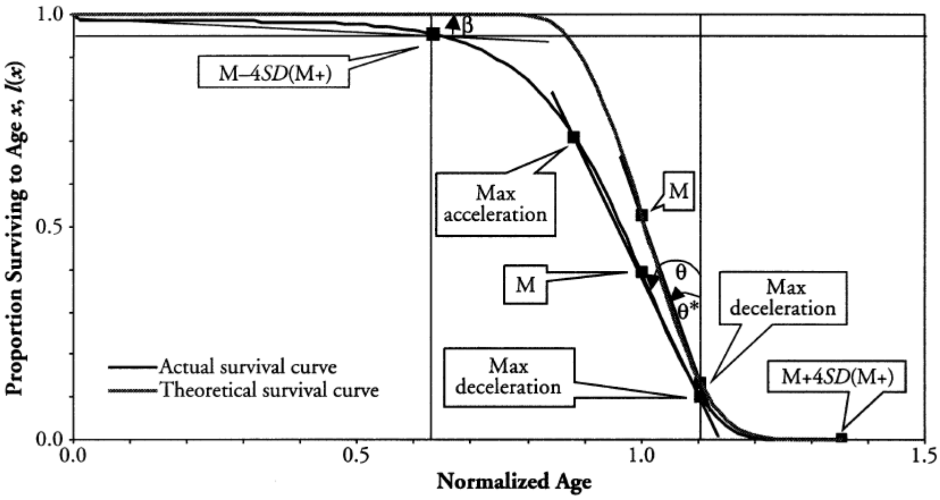


Figure 6. Three-dimensions of Mortality Compression (Source: Cheung et al., 2005)

From the brief review of the past work, we can see that the current studies of mortality compression all rely on the values of life tables. However, usually life tables are developed through a graduation process. The graduation methods and assumption (e.g., the ultimate age or the highest age attained) have noticeable influences on life table values, but the methods and assumptions are quite different depending on the users. Ideally, studying mortality compression should not depend on the choices of methods and assumptions. In the next section, we propose using the raw data, instead of the graduated values, to evaluate the mortality compression.

3. Proposed Measurements and Empirical Studies

In this section, adapting the idea from Cheung et al. (2005), we shall first introduce the modified measurements based on the raw data, instead of the graduated values. Then, the modified approach will be applied to the data from Japan, Sweden and the U.S. We shall first define the notation before introducing the proposed approach.

The number of deaths between age x and x+1 is denoted by d_x in life tables, which can be expressed as $d_x = l_x \cdot q_x$, where l_x is the number of survivors aged x, q_x is the (conditional) probability of an individual age x who will die before age x+1. If we use the survival probability p_x, or $p_x = 1 - q_x$, then the (unconditional) probability of an individual age 0 who will die at ages between x and x+1 is:

$$ {}_x p_0 \cdot q_x = q_x \cdot \prod_{y=0}^{x-1} (1 - q_y) \tag{1} $$

We can also derive the number of deaths via $d_x = l_0 \cdot {}_x p_0 \cdot q_x$, where l_0 is the number of survivors at age 0 (i.e., radix, usually set to be 100,000). Because l_0 is a fixed value, the new expression of d_x does not depend on the assumption ultimate age and the mortality rates of age greater than x. Then, we can still use the values of d_x to evaluate mortality compression without being influenced by graduation methods and assumptions. Note that these life table functions, i.e., q_x, p_x, d_x, l_x, are all defined in a period (not cohort) basis.

As expected, using the raw data, the age specific mortality rates q_x in (1) would not be smooth and are likely to have fluctuations, and so do the numbers of deaths d_x derived from q_x. Therefore, the survival and mortality curves will not be smooth as shown in Figure 4, and the measurements to evaluate mortality compression are not easy to define. For example, it is difficult to compute the angles of horizontalization β and verticalization θ in Cheung et al. (2005). Thus, we need to modify measurements for evaluating the mortality compression.

In specific, we choose the following five measurements:

(i) The mode age (denoted by M), which has the largest values of d_x, should be highly correlated with life expectancy.

(ii) The probability of premature death is defined as P(0 ≤ X ≤ m), m = 10, 20, 30, 40, 50, where X is the random variable of age-at-death. The meaning of this measurement is similar to the horizontalization.

(iii) The smallest number of ages covers the probability α of deaths, or $C_\alpha = P(x \le X \le x+\delta) = \alpha$, where $\delta > 0$. This is the same measurement used in Kannisto (2000) and it is closely linked to the verticalization.

(iv) The variance of age distribution for deaths σ^2.

(v) The probability of survival beyond a high age, or $P(X > M + k\sigma)$, where $k > 0$.

The first three measurements can be treated as the three-dimension measurements in Cheung et al. The last two can be used to evaluate the life limit and check if the age distribution for deaths satisfies normal distribution assumption.

It should be noted that, in addition to the usual formula for computing sample variance, there are other ways for computing the variance of age distribution for deaths. As mentioned by Kannisto (2000), the numbers of deaths on the right side of the mode do resemble the shape of the probability density function of a normal distribution. Plugging the probability density function of a normal distribution, we have

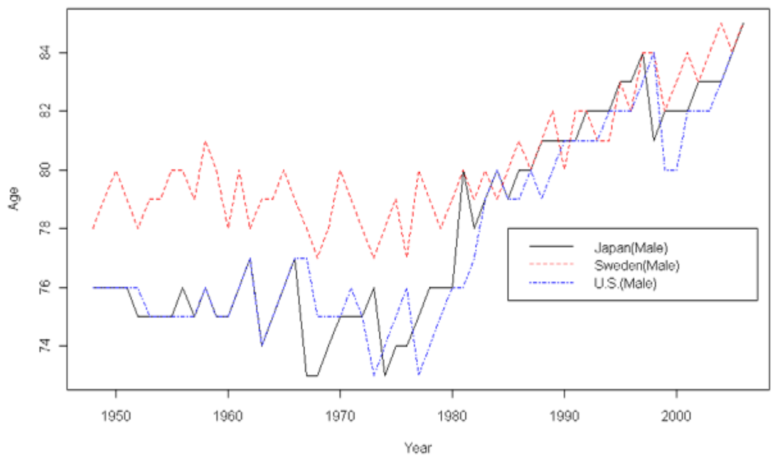
$$\frac{d_x \cdot d_{x+2}}{(d_{x+1})^2} = \frac{(p_{x-1} \cdot q_x) \times (p_{x+1} \cdot q_{x+2})}{(p_x \cdot q_{x+1})^2} = \exp(-\frac{1}{\sigma^2}). \tag{2}$$

In other words, if the ratios of numbers of deaths in (2) are not constant of age, then the assertion by Kannisto is not true. If the ratios are fixed, then the weighted least squares can be used to compute the variance, after taking the logarithm on both sides of (2).

Before showing the empirical results of data from Japan, Sweden and the U.S., we first compute the proposed measurements under two graduation methods. Table 1 shows the results based on 1999-2001 Taiwan data. We can see that using different graduation methods of the standard deviation σ estimate has a larger difference for the female data, and the difference in σ estimate is about 9 percent. Suppose the same data set is used. We would still observe differences if different graduation methods are used, which indicates the potential problems for using the graduated values.

Table 1. Two Graduation Methods in Taiwan 1999-2001

	Male		Female	
	Whittaker	Gompertz	Whittaker	Gompertz
M	83	83	85	86
σ	6.77	6.68	6.35	5.82
Life Expectancy	73.64	73.52	79.44	79.32



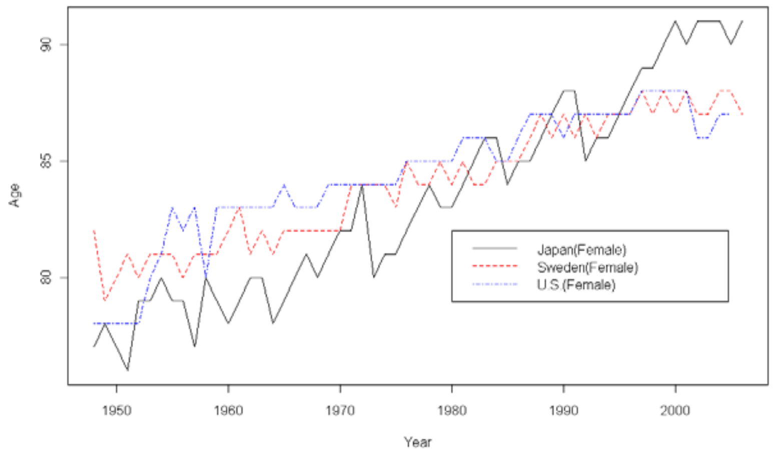


Figure 7. The Modes of Distribution of Age-at-death

The mortality data of Japan, Sweden and the U.S. are all from the Human Mortality Database (HMD). The data periods of these three countries are different, according to the data availability, where Japan is 1947-2006, Sweden is 1901-2007, and the U.S. is

1933-2005. To compare these three countries, only the time period 1947-2006 is used. We will first discuss the mode ages with the largest values of d_x for these three countries (Figure 7). The modes of males are near constant before the 1980s and increase linearly after that. Also, the modes of Sweden are obviously larger before 1980 and now the three countries have about the same mode ages. The modes for females show a similar pattern but their increments are more stable. Japanese females appear to have larger modes than those in Sweden and the U.S. Heuristically speaking, the mode can be treated as life expectancy and the trend of the mode indicates that the human life continues to extend.

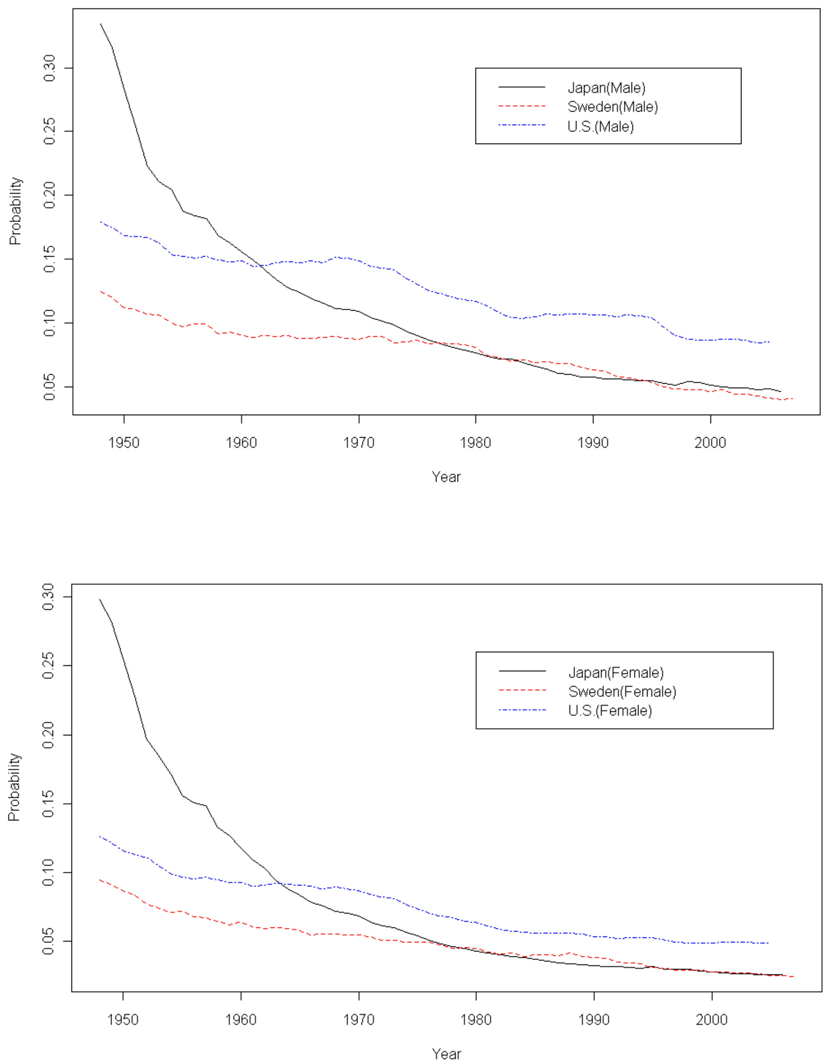


Figure 8. The Probabilities of Death Before Age 50

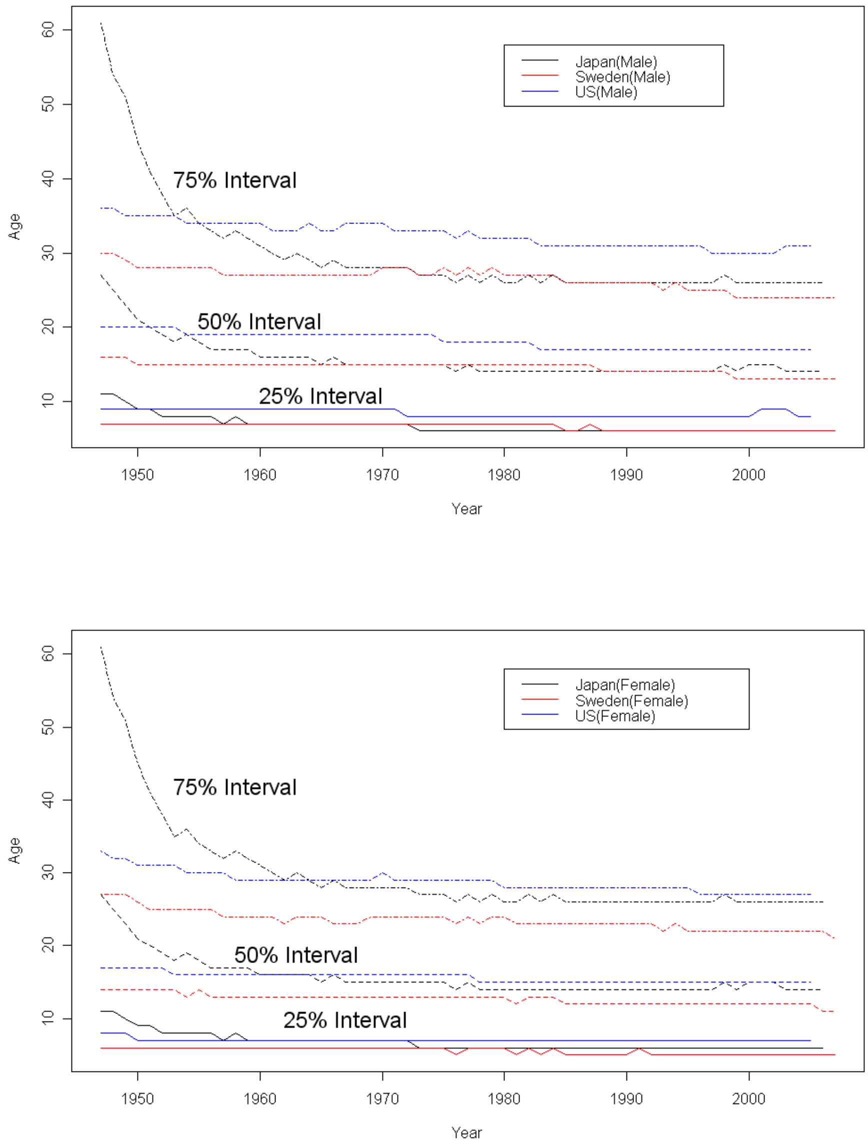
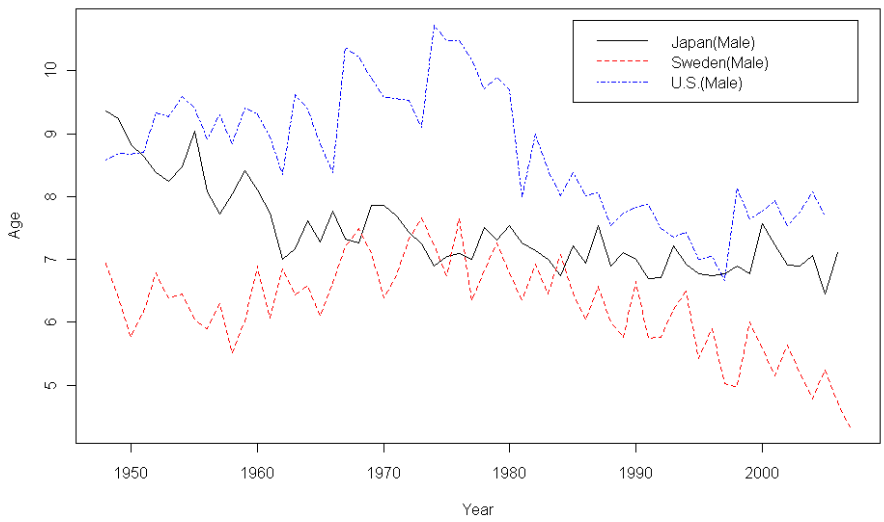


Figure 9. The Shortest Age Intervals of 25 Percent, 50 Percent and 75 Percent

The ideas similar to the horizontalization and verticalization in Chueng et al. (2005), i.e., the probability of premature death and the smallest number of ages covers the probability of deaths, are shown in Figures 8 and 9. Figure 8 shows the probabilities of dying before age 50. The probabilities of death of Japanese males and females are more than 0.30 in the 1950, the highest among three countries, and have the largest decreasing rates. The probabilities of death in Sweden and the U.S. seem to decrease constantly, and the differences between these two countries are about 5 percent. The probabilities of death of Japan and Sweden are about the same since 1980. Also, the probability of death in the U.S. is the largest after 1970, and this is highly correlated to the shortest life expectancy. Figure 9 is the smallest number of ages covering certain probabilities of deaths, and this is similar to the idea of verticalization. No matter if the probability is 25 percent, 50 percent or 75 percent, the age ranges tend to decrease gradually. This indicates that the distribution of deaths is more concentrated, which seems to support the mortality compression. Also, the age ranges of the U.S. are the widest among the three countries.



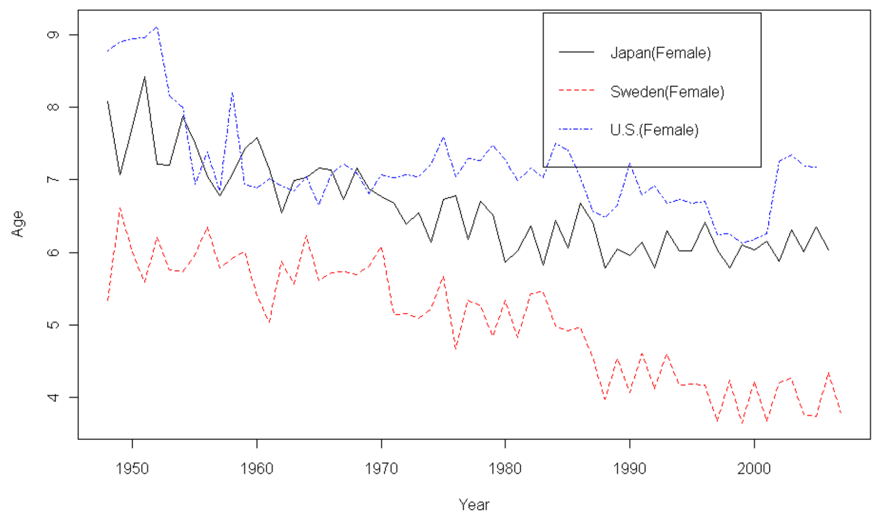


Figure 10. The Standard Deviations of Death Age

Based on the preceding measurements, life expectancy seems to extend gradually and the mortality compression tends to hold. The standard deviation of age distribution for deaths will be used as a double check for the mortality compression. However, instead of following the method of Cheung et al. (2005), we use raw data and apply the weighted least squares on (2) to compute the variance. Note that, the differences are less than 10 percent if applying the method of Cheung et al. Figure 10 shows the standard deviations and there exist noticeably large fluctuations since the raw data are used. But unlike the result of Hong Kong in Cheung et al., the standard deviations do not always decrease. Those of Japan and the U.S. seem to level off, while those of Sweden do decrease annually. We do not have concrete evidence for supporting the mortality compression.

The probabilities of surviving beyond very high age do not support the mortality compression either. Figure 11 shows the survival probabilities beyond the mode plus one standard deviation and two deviations of age distribution. The ages of these two bounds are approximately 95 and 100, and thus can be treated as "very high age" survival probability. (It is like the elderly of age beyond 85 defined as "oldest-old.") We can use these two survival probabilities as proof of whether life extension reaches a limit. Because of the insufficient sample of the elderly, the survival probabilities have larger fluctuations (about 40 percent more). Interestingly, the survival probabilities of the three countries are very close to each other. In general, the survival probabilities beyond $M+\sigma$ of males and females are about 10~12 percent and 11~13 percent, respectively, and those beyond $M+2\sigma$ of both males and females are about 1~2 percent.

Although the measurements of horizontalization and verticalization support mortality compression, the standard deviation and the survival probability beyond very high age provide another possibility for future life expectancy. In particular, the survival probabilities beyond very high age and the standard deviations of age distribution for Japan and the U.S. all look constant. Together with if the mode age (or life expectancy) continues to increase, it seems that life expectancy having a limit is still an open question. We tend to believe life is without a limit, where the survival curve is likely to move to the right, as shown in Figure 12.

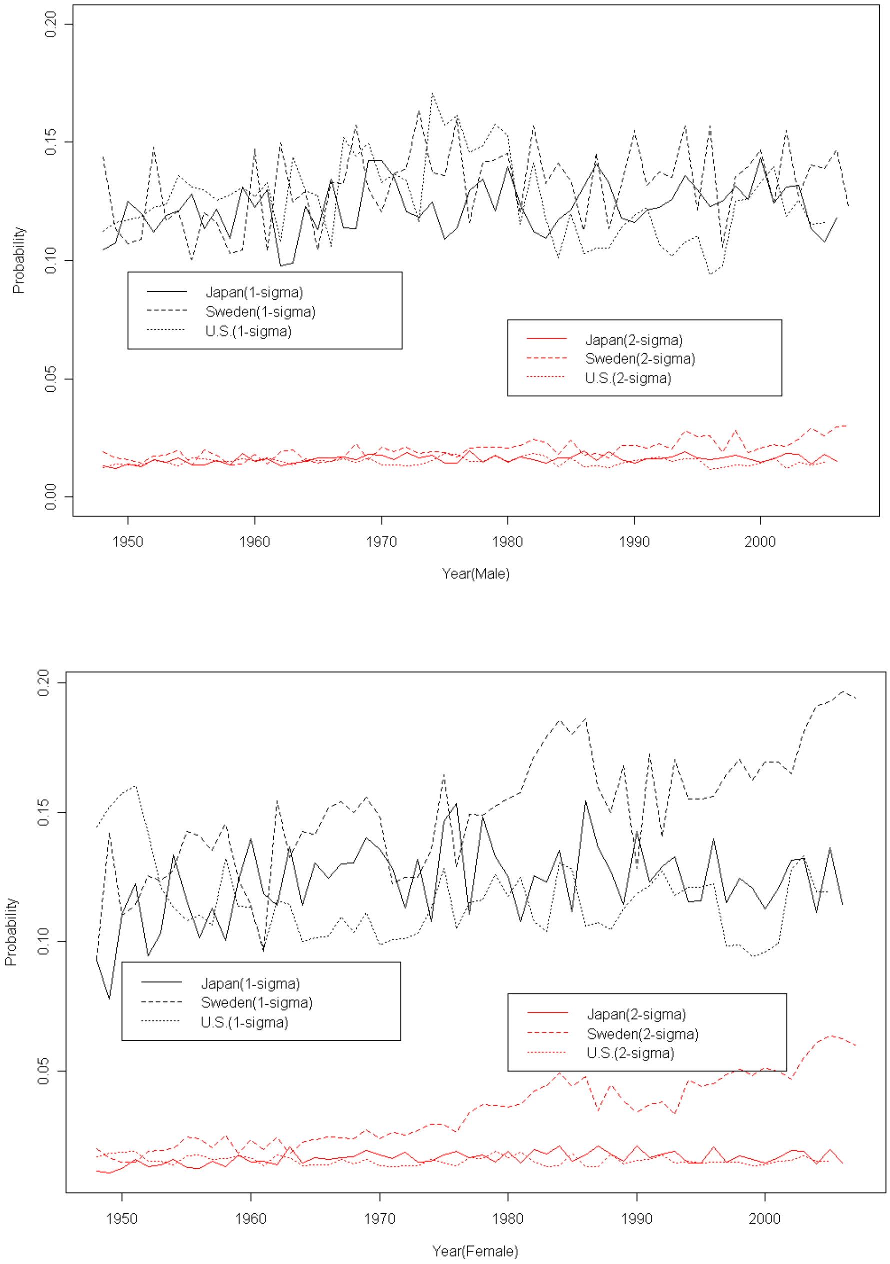


Figure 11. The Probability of Survival Beyond a High Age

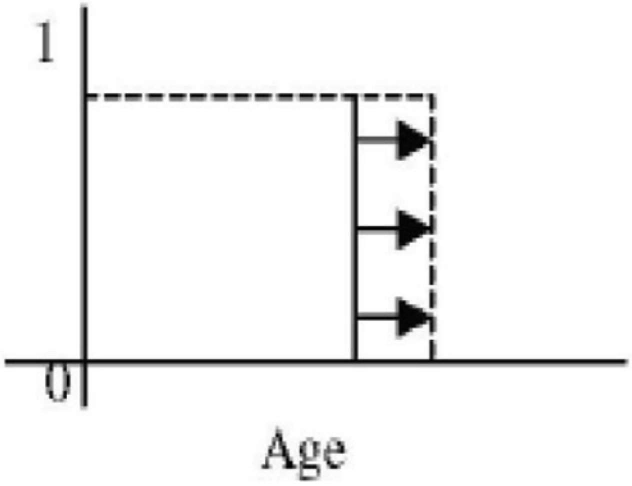


Figure 12. The Theory of Life Without a Limit

4. Implications to Annuity Products

The continuing reduction in mortality rates has been creating difficulties in designing annuity products. In the past, people tended to believe that life expectancy would stay constant or reach a limit. For example, in the 1960s, the consensus was that human beings would likely not live beyond age 85. However, just after 40 years, the life expectancies of some countries have already surpassed 85 years, such as happened in Japan and Hong Kong in 2007. The assertion that life expectancy is a constant or has a limit now is questioned by many researchers.

Instead, it seems life expectancy is more likely to continue increasing, at least for the near future. To address the increasing life expectancy, stochastic mortality models have become a popular tool in calculating the value of annuity products. The Lee-Carter model (Lee and Carter, 1992) and the reduction factor model (Continuous Mortality Investigation, 1999) are two of the famous mortality models. Life expectancy under these models also will continue to increase. Unlike the previous studies, we used the raw data to verify mortality compression and found there are no obvious signs of converging life expectancy. On the contrary, the results support that life expectancy is going to extend, like in the stochastic models.

Although it is assumed that life expectancy would extend in the stochastic models, there are still doubts for using them to model annuity products. For example, in the Lee-Carter model, where the age-specific mortality rates are assumed to reduce exponentially annually, the reduction rate plays an important role in pricing. If the reduction rate is underestimated, the annuity products will be underpriced. However, there are insufficient data of the elderly, and this raises doubts in estimating reduction rates for the elderly, especially for the oldest-old. Because there are insufficient samples for people 90 and older, modeling mortality rates of the elderly usually relies on extrapolation methods, such as the Gompertz law. This means we are not sure about the right tail of the survival distribution and depend solely on assumptions to price annuity products. Thus, it is difficult to evaluate the risk of using stochastic mortality models to price the annuity products.

If the mortality compression is true, we can use it to modify mortality models and evaluate the risk. Unfortunately, we found that mortality compression is not always true. If the variance (i.e., risk) of the death age distribution is decreasing, we have more confidence in designing annuity products. But, from the historical data from Japan and the U.S., the standard deviations seem to be constants. Besides, the survival probabilities beyond ages 95 and 100 (approximately, $M+\sigma$ and $M+2\sigma$) also behave like constants. This indicates there is a non-negligible probability for the right tail of the survival distribution that is still unknown or without enough observations.

In other words, the study of mortality compression does not help to relieve doubts in using the mortality models. Instead, the results in this study might lead to another possibility in designing annuity products. For example, since the survival probability beyond $M+2\sigma$ is about 2 percent, this suggests that the death age distribution is well studied on the left hand side of $M+\sigma$ (or even $M+2\sigma$). But, for the age distribution beyond $M+2\sigma$, fewer data are available and it is difficult to evaluate the survival probability, especially to predict the future survival probability.

One of the possible approaches for dealing with longevity risk is to concentrate on the part with more confidence. For example, we can focus more on products like annuity-certain, and design the annuity payable up to the age of $M+2\sigma$ and consider other tools to cope with ages beyond $M+2\sigma$. Note that the age of $M+2\sigma$ is about 100 years old and there is about 2 percent of probability of surviving beyond this age. It would be plenty for most people.

There are several ways to tackle the coverage of survival beyond the age $M+2\sigma$, but first we need to decide the role of annuity products in planning one's retirement life. If the annuity products are not the primary source of income, then the coverage up to the age $M+2\sigma$ probably is enough. If the insured relies solely on annuity products, then we need to deal with estimating the probability of surviving beyond age $M+2\sigma$. However, up to today, there is not enough data to estimate this probability.

5. Discussions

Since the idea of mortality compression and rectangularization was proposed in 1980, many studies have tried to verify the idea and check if it can be linked to exploring the theory of life limit. Although it is believed that life expectancy will increase in the future, the theory that there is a limiting age to life is preferred. However, in verifying the mortality compression, most studies use graduated mortality rates from life tables instead of the raw data. Of course, the graduated values usually are very smooth, and it is easy to capture properties of survival curves by visualization. But the graduation might distort the nature of mortality rates. Since the numbers of observations for the elderly are seldom sufficient, their graduated mortality rates are likely to be very different from the true values and cause false decisions. The data problem of the elderly possibly is the most challenging for the study of human longevity.

The problem is even more obvious if the focus is on areas with few populations. For example, Cheung et al. (2005) studied mortality compression in Hong Kong, based on the whole life tables, where three-year death records are used. However, as shown in our study, techniques of constructing life tables may have significant influences on mortality rates and possibly on the conclusion of mortality compression. Thus, we propose measurements to evaluate mortality compression based on raw data to reduce the influence of graduation. Still, insufficient samples of the elderly cause fluctuations in the measurements, which are especially observable in the case of Sweden since it has the smallest population.

We applied the data from Japan, Sweden and the U.S., and reached similar conclusions as in Cheung et al. We found the age with the largest number of deaths (mode M) increases annually, and the probability of premature death (horizontalization) and the smallest number of ages covers the probability of deaths (verticalization) decrease annually. But for the issue of whether life expectancy has a limit or mortality compression, our findings are not the same as that in Cheung et al. The standard deviations of the death age distribution show different patterns. Like in Hong Kong, the standard deviations in Sweden seem to decrease gradually but those in Japan and the U.S. look like constants, which means there are inconclusive evidences for supporting the mortality compression and the theory of life with a limit.

In addition, we found the probabilities for surviving beyond the age of $M+\sigma$ or $M+2\sigma$ (i.e., very high age) remain constants in the three countries and both sexes. This can be used to act against the theory that life expectancy is reaching a limit. Note that, although the standard deviations show different patterns, the sum of the mode age and standard deviation also increases (like the mode M). Together with the result that the probability of survival beyond a very high age is constant, it seems life expectancy will continue to increase. Li et al. (2008) also had a similar conclusion. They proposed a model using the idea from extreme value theory and found that the human lifespan is not approaching a limit, based on the raw data from the HMD.

The prolonging of life has increased the complexity of designing annuity products and the insufficient data of the elderly even makes the situation more complicated. The lack of data escalates the risk in designing annuity products, due to the variances of mortality estimates. The findings in this study might provide some possible alternatives for the problem. For example, we can use the result that the probability of survival beyond very high age is close to a constant, to reduce the risk to annuity products. A possible approach is to design annuity-certain products and let the annuity be payable up to a very high age (e.g., M+2σ). Then, we can concentrate on the methods for dealing with surviving beyond the very high age (about 2 percent of probability).

Our study wants to show that the data quality is crucial in exploring the mortality compression. Most of the previous work used graduated data (e.g., mortality rates and death numbers) from life tables. These numbers follow the assumption of stationary population, with same radix $l_0 = 100{,}000,$ meaning that 100,000 newborns every year. The proposed method intends to avoid the stationary population assumption and to use the population size to adjust the estimation (weighted least squares). Note that all countries involved in this study, official population counts are available at least up to age 90 (Jdanov et al, 2008). The HMD makes adjustment for the mortality rates over age 90 by the survivor ratio method (Thatcher et al, 2002). Since the population over age 90 is not much, the adjustment has a small impact on the estimation of modal age and standard deviation σ (Yue, 2002).

The results we found are for Japan, Sweden and the U.S., and it does not mean they will apply to other countries. There are reasons for choosing these three countries as a pilot study. These countries have larger populations, longer life expectancies, and especially good data quality and longer data periods. We applied the proposed measurements to Taiwan data and found some funny results that are not easy to interpret. For example, the standard deviations of the death age distribution are decreasing but the probabilities for surviving beyond the age of M+σ or M+2σ are increasing. We will continue exploring the mortality compression on other countries.

In this paper, we proposed measurements based on the raw data to evaluate the mortality compression, to reduce the influence of graduation in life tables. However, since the values are not graduated, the results generally are not very smooth and fluctuate a lot. The fluctuations are particularly obvious for the case of Sweden and would also be for areas with small populations. We can combine the single-year results to smooth the measurements (such as a three-year or a five-year moving average). But for areas with populations less than in Sweden (e.g., Hong Kong), measurements from the single-year data still will be not good enough and multi-year data might be needed. If this is the case, then the proposed measurements need to be modified.

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