A Ratio Method for Old Age Mortality Projection Based on Incomplete Data: The Case in Taiwan

台灣地區高齡人口的死亡率推估：
比值法在不完整資料下的應用

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Abstract

Mortality projection for the elderly plays a critical role in designing welfare policies for them. Most prevailing projection methods perform well when there is sufficient mortality information. However, the projection power of these methods is uncertain when the mortality profile is incomplete. In this paper, we propose a ratio method, which includes techniques used in regression and time series analyses that utilize the relationship between the old-age group and their younger counterpart to compensate for the lack of mortality information on the former. A simulation demonstrates how this method reduces projection errors. In addition, we compare this new method to three prevailing projection methods (Lee-Carter, SOA, and Pollard). Based on the mortality data on the elderly in Taiwan between 1950 and 1997 and using the data between 1950 and 1992 as pilot data and that between 1993 and 1997 as test data, we evaluate the prediction power of these methods and find that the ratio method is the best for predicting female mortality rates in terms of the absolute and squared errors.

Key Words: projection, old age mortality, incomplete data, simulation, ratio method

摘 要

死亡率的推估在設計老年人的福利政策上扮演一個關鍵性的角色。當各年齡死亡資料完整時，常見的死亡率推估方法大多都預測良好；但若高年齡組的

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死亡資料不完整時，這些方法的預測能力則不能確定。鑑於相關文獻十分缺乏，我們乃於本文中提出一個「比值推估法」，以降低推估的不確定性。此推估法結合迴歸模型和時間序列模型的方法，估計出各高年齡組（通常資料較不充足）死亡率與某一參考年齡組死亡率之間的比值關係；本文也以電腦模擬討論比值法如何減少推估的誤差。另外，我們以台灣 1950 年到 1997 年的高齡人口死亡資料為分析對象，將比值法與其他三種最為常用的方法（Lee-Carter, SOA 及 Pollard）的推估結果比較，發現在絕對和平方誤差為判斷依據時，比值法在預測女性死亡率上均為最好的方法。

關鍵字：推估、高齡死亡率、不完整資料、模擬、比值法

I. INTRODUCTION

A major concern of policymakers in Taiwan, as in many other developing countries is the population aging of the society. From 1951 to 1997[1], life expectancy in Taiwan rose from 53.4 to 72.0 years for made and from 56.3 to 77.9 years for female (see Table 1 for details). On one hand, prospects of longer life are viewed as a positive change for individuals and as a substantial social achievement. On the other hand, those prospects have led to concern over their implications for public spending on old-age support. Recently, Taiwan is in process of pension reform and starting to implement several social welfare programs regarding the elderly. Therefore, making reliable mortality projections for the elderly population is a top priority.

Based on varies assumptions on the population, there are many methods to forecast mortality (Tuljapurkar and Boe, 1998). Among them, the SOA (Society of Actuaries) method, Lee-Carter method and Heligman-Pollard method are the most sophisticated and widely used forecasting methods. However, applying these methods to project the mortality for the elderly in Taiwan is not always successful.

One major problem is due to the incomplete data of the elderly mortality, a common problem existing in most developing countries. "Incomplete data" are mainly attributed to the lack of observations in advanced age groups (i.e., aged 85 and over). Most of the time, those scattered observations in the advanced age groups are categorized as one big age group (i.e.,

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[1] Note that there were also a number of substantial reductions in Taiwan mortality rates in years before 1950. However, for the sake of the data homogeneity, we only consider the mortality rates between 1950 and 1997.

85+) such that the age-specific mortality projection for these groups is rather difficult. Note that there are detailed records for ages higher than 85 in recent years, for example, in 1999, the group 85+ is divided into 85-89, 90-94, 95-99, and 100+. However, the numbers of observations in these advanced age groups (85+) are much smaller than other elderly groups, which may reduce the precision of overall estimation. Thus, in this study, we only consider the mortality data for ages between 60 and 84 to evaluate different projection methods.

Table 1 The Life Expectancy at Birth of People in Taiwan in Selected Year

Year	Male	Female	Year	Male	Female
1951	53.38	56.33	1991	71.83	77.15
1956	60.40	63.25	1992	71.79	77.22
1961	62.30	66.76	1993	71.62	77.59
1966	65.18	69.74	1994	71.83	77.82
1971	67.19	72.08	1995	71.73	77.79
1976	68.70	73.59	1996	71.94	77.77
1981	69.74	74.64	1997	71.99	77.85
1986	70.97	75.88			

Source: Abridged Life Tables of Taiwan-Fukien Area, Department of Statistics, Ministry of Interior, R.O.C.

In this paper, we propose a new method that can accommodate the incomplete data more effectively to forecast the mortality rates. Our method is distinct from other methods by accounting for the correlations between a chosen age group (ages 60-64 as the base group) and age groups of the elderly. Since these correlations are very high (For example, the correlation coefficient between the mortality rates of ages 60-64 and 65-69 is slightly over 0.95 in Taiwan), we can use the mortality information contained in the base group to improve the mortality prediction of the elderly. This method has its advantage, in particular, over other forecasting methods when the number of observations in the age groups of interest is very small and that of the base group is relatively large.

In the rest of this paper, we first describe the mortality data of Taiwan and their characteristics. Next, we present the idea of our method and introduce three well-known projection methods. Then we apply these methods to fit the mortality rates of the elderly in Taiwan and evaluate their performance. In the last section we discuss the implications and limitations of our method, and propose some possible extensions for future study.

II. DATA

The data used in this study are from *Taiwan-Fukien Demographic Fact Book Republic of China* between 1950 and 1997, published by Department of Statistics, Ministry of Interior, R.O.C. Since we are interested in the mortality profiles of the elderly, only the data of ages 60-64 and higher quinquennial age groups (such as 65-69) are used. The characteristics of this data set can be shown visually and for example, Figure 1 shows the mortality rates of male elderly during 1950 to 1997. Before the year 1958, the highest recorded age group of mortality rates is ages 70 and over (i.e., 70+); the highest recorded age group is 80+ between 1959 and 1970; the highest recorded age group is 85+ after 1970. In other words, for the male and the female each, there are 48 observations available for the groups 60-64 and 65-69 between 1950-1997. However, only 39 observations are available for the groups 70-74 and 75-79, and 23 observations for the group 80-84. This incomplete structure of mortality data (or, incomplete data, for short) is due to the size of those age groups being very small in the early days.
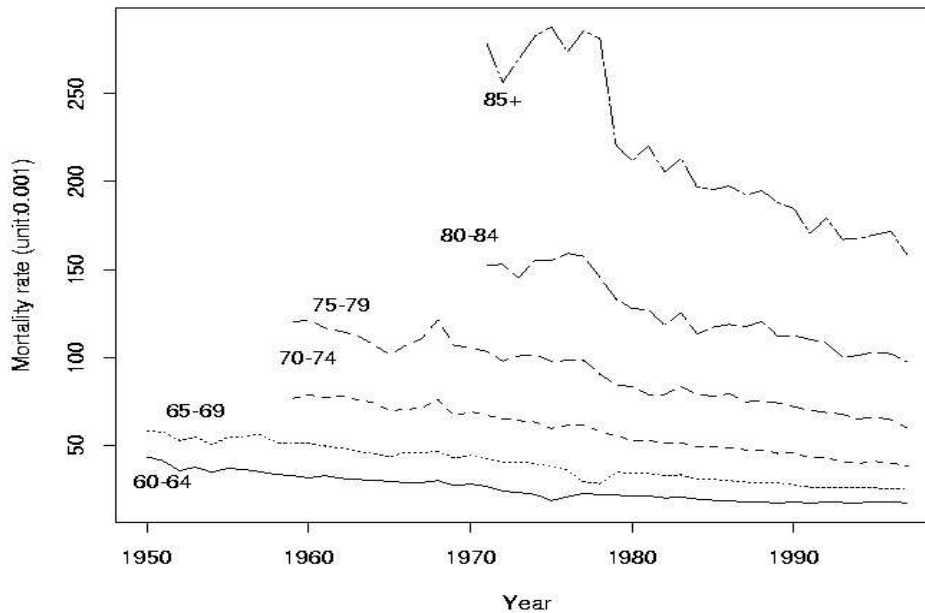

Figure 1 Mortality Rates of the Elderly Male in Taiwan

Unfortunately, this incomplete data structure will weaken the projection power of

certain existing projection methods (e.g., Lee-Carter method). To account for these problems, we propose a new projection method in next section. In addition, we briefly introduce three existing methods and discuss the similarities and differences among them.

III. FOUR METHODS

In the beginning of this section, we describe the process of applying the ratio method and illustrate its underlining idea of accounting for the incomplete data problem.

1. The Ratio Method

In the ratio method, the mortality rates of ages 60-64 are used as the base to predict those of the elderly (i.e., ages 65-69, 70-74, 75-79, 80-84). The process of applying the ratio method is as follows: First, we construct a model for the mortality rates of the group 60-64. (The projection of the mortality rates of ages 60-64 is similar to that in the SOA method, which will be discussed later.) Then compute the ratio of mortality rates

$$R_x(t) = \frac{_5q_x(t)}{_5q_{60}(t)}, \tag{1}$$

for ages $x=65,70,75,80$. Since the ratios $R_x(t)$'s are not constants over time, we apply time series models to estimate. The prediction of future mortality rates of ages 65-69, 70-74, 75-79, and 80-84 can be obtained according to the following formula,

$$_5\hat{q}_x(t) = \hat{R}_x(t) \times {}_5\hat{q}_{60}(t),$$

where $\hat{R}_x(t)$ and $_5\hat{q}_{60}(t)$ are the predicted values of the ratios and the mortality rates of group 60-64 at time t, respectively. The idea of the ratio method is similar to *Control Variate* (a method related to *Variance Reduction* in simulation). For details of the variance reduction techniques, see Chapter 5 in Ripley, 1987. We use the following example to demonstrate how and why the ratio method works. Let (X,Y) denote the mortality rates of age group x_0 and y_0 (i.e., x_0 is the group 60-64 and y_0 is the group 65-69, or higher ages), respectively, at a certain point of time t. We assume the (X,Y) are random variables from a bivariate normal distribution, i.e.,

$$\binom{X}{Y} \sim N\left(\binom{\mu_X}{\mu_Y}, \begin{pmatrix} \sigma_X^2 & \rho\sigma_X\sigma_Y \\ \rho\sigma_X\sigma_Y & \sigma_Y^2 \end{pmatrix} \right).$$

Assume that $(x_i, y_i),\ i = 1,...,n$, and $x_j,\ j = n+1,...,n+m$, are available. Because there are more observations for x's and positive correlation exists between X and Y, it is natural to use x's to improve the accuracy in estimating μ_Y. For example, we can revise the original estimate $\hat{\mu}_Y(1) = \sum_{i=1}^{n} y_i/n$ by using

$$\hat{\mu}_Y(2) = \bar{x}_{n+m} \times \frac{1}{n}\sum_{i=1}^{n} \frac{y_i}{x_i},$$

where $\bar{x}_{n+m} = \sum_{i=1}^{n+m} x_i/(n+m)$ can be treated as the estimate of $_5q_{60}(t)$ and y_i/x_i can be treated as the ratio $R_x(t)$. Let ($\mu_x, \mu_y, \rho, \sigma_x, \sigma_y$) = (2, 3, 0.95, 0.2, 0.3). Based on 10,000 simulations, the estimates of μ_y are as following: (Numbers inside the parentheses are standard deviations.)

Table 2 Examples of Variance Reduction in Applying The Ratio Method

	$\hat{\mu}_Y(1)$	$\hat{\mu}_Y(2)$
$n = 34$	3.000408	3.002138
$m = 9$	(.0026533)	(.0021510)
$n = 23$	3.000246	3.000802
$m = 20$	(.0038690)	(.0023129)

Note that $n+m = 43$ can be treated as the number of observations available for the group 60-64 between 1950-1992 and n is the number of observations available in the elderly groups, where $n = 34$ corresponds to ages 70-74 and 75-79, and $n = 21$ corresponds to ages 80-84. We can see that $\hat{\mu}_Y(2)$ has smaller standard deviations and larger m would give smaller standard deviation.

More simulations are conducted to further investigate the effect of variance reduction in using $\hat{\mu}_Y(2)$. The numbers in Table 3 are the simulation result based on 100 simulation runs, and each run is based on 1,000 replications. The number in each cell is the number of times that $\hat{\mu}_Y(2)$ has smaller variance than $\hat{\mu}_Y(1)$ in 100 simulation runs. Adapting the idea of testing hypothesis, numbers larger than 60 (or smaller than 40) indicate that $\hat{\mu}_Y(2)$ has smaller (or larger) variance, with significance level $\alpha = 0.05$. When the correlation between

X and Y is stronger, $\hat{\mu}_Y(2)$ performs better. Also, the smaller σ_X is, the smaller variance $\hat{\mu}_Y(2)$ has. In our case, we would expect that the ratio method has better performance in younger elderly groups, since the correlations between these groups and ages 60-64 are stronger than that of older elderly and ages 60-64.

Table 3 Number of Times $\hat{\mu}_Y(2)$ **Has Smaller Variance over 100 Simulations**

(Each simulation is calculated based on 1,000 replications.)

$(\sigma_X,\sigma_Y)=$	(0.2,0.2)		(0.2,0.4)		(0.4,0.2)	
$(n,m)=$	(20,10)	(20,5)	(20,10)	(20,5)	(20,10)	(20,5)
$\rho=0.95$	100	100	100	100	0	0
0.9	100	99	100	100	0	0
0.8	72	70	100	100	0	0
0.7	7	16	100	100	0	0
0.6	0	1	100	98	0	0
0.5	0	0	93	81	0	0

Tuljapurkar and Boe (1998) and Pollard (1987) made a critical assessment of knowledge about mortality change and reviewed methods of mortality projections. Among all methods, the following three methods are used most often without directly accounting for the pattern of deaths by different causes: the method used by the SOA, Lee-Carter's method (Lee and Carter, 1992), and Heligman-Pollard's (Heligman and Pollard, 1980) method. We will introduce these methods in this order.

2. The SOA Method

The SOA method can be categorized to projection by extrapolation, which is the simplest and most widely used method, and is of the form

$$_n q_x(t+k) = (g_x)^k {}_n q_x(t) \ ,\tag{2}$$

where $_np_x(t)=1-_nq_x(t)$ is the probability of survival from age x to age $x + n$ at time t and g_x (usually $0 < g_x < 1$) is the rate of change in that probability (which takes account of improvements in mortality at age x over time). The SOA used this method for individual annuity valuation and the Institute of Actuaries (London) also used a similar method for building annuitant tables.

To determine the value of g_x , we usually take the natural logarithm of (2) and apply regression analysis:

$$\ln {}_n q_x(t+k) = \ln {}_n q_x(t) + k \ln(g_x),\tag{3}$$

by assuming $g_x = \exp(\beta_1(x))$,

$$\ln {}_n q_x(t+k) - \ln {}_n q_x(t) = \alpha_1(x) + \beta_1(x) \ k \ ,$$

where $\alpha_1(x)$ and $\beta_1(x)$ are constants with respect to time t.

Note that for different ages x and y, from (2) and (3),

$$\ln\left(\frac{{}_n q_x(t+k)}{{}_n q_y(t+k)}\right) - \ln\left(\frac{{}_n q_x(t)}{{}_n q_y(t)}\right) = \alpha_1(x) - \alpha_1(y) + k \ln\left(\frac{g_x}{g_y}\right) \ .\tag{4}$$

This implies that the logarithm of the mortality ratio for different age groups also has a linear relationship, which is different from the ratio method. If $\ln(g_x/g_y)>0$ then the age group $[x, x + n]$ has a better mortality improvement than the age group $[y, y + n)$ over time, and vice versa.

3. The Lee-Carter Method

Lee and Carter (1992) proposed a remarkably simple model for U.S. morality projections. They assume that

$$\ln m_{xt} = \alpha_2(x) + \beta_2(x) k_t \ ,\tag{5}$$

where m_{xt} is the central death rate for age x at time t. Here k_t can be viewed as a stochastic process and is fitted accurately by the following time series model:

$$k_t = k_{t-1} - z + \varepsilon_t \ ,\tag{6}$$

where z is a constant average rate of decline and ε_t is a random term whose statistical properties are estimated from the data.

To solve for the parameters α_2's, β_2's, and k_t, usually *Singular Value Decomposition* (SVD) is used. However, when $\ln m_{xt's}$ do not form a complete matrix (for example, the mortality records of the older elderly are missing or not complete) and SVD cannot be applied, Lee and Carter suggested a method of approximation.

In forecasting U.S. death rates, Lee and Carter found that k_t is close to a linear function of time t and declines at an average rate of 0.365 per year from 1900 to 1989. Lee and Nault (1993) and Wilmoth (1996) applied the Lee-Carter method to forecast Canadian and Japanese mortalities, respectively. Note that if k_t is a linear function of time t, the Lee-Carter model is actually similar to the SOA model (by comparing (3) and (5)), except that death rates (q_x) are used in the SOA model and central death rates (m_x) are used in the Lee-Carter model.

4. The Heligman-Pollard Method

Heligman and Pollard (1980) proposed an eight-parameter model to predict the curve of death rates in Australia:

$$\frac{p_x}{q_x} = A^{(x+B)^C} + D\exp\left(-E(\ln x - \ln F)^2\right) + GH^x,\tag{7}$$

where $A^{(x+B)^C}$ represents the decreasing trend of death rates in young ages, $D\exp\left(-E(\ln x - \ln F)^2\right)$ is the death rate around age 20 (with a big hump due to accidental deaths), and GH^x shows the exponential decreasing pattern in adult mortality rates. Unlike the previous models, this model has a parametric function form. Similar to the Lee-Carter method, these eight parameters can be solved by time series methods.

However, although the Heligman-Pollard method was intended to increase the precision of projection (e.g. compared to Lee-Carter's), comparisons to the Lee-Carter forecasts have not been done. Also, this method is rarely used due to the complexity of estimation for eight parameters.

IV. DATA ANALYSIS

As described in Section 2, our data analysis is based on Taiwan mortality data by age and sex during the year of 1950 to 1997. We apply the methods discussed in Section 3 to fit mortality data from 1950 to 1992, and evaluate the performance of these methods by using the data from 1993 to 1997. Note that since the ratio method, the SOA method, and the Heligman-Pollard method use the death rate and the Lee-Carter method uses the central death

rate, we apply the *Uniform Distribution of Deaths* (U.D.D.) assumption, i.e.,

$$_nq_x = \frac{n_n m_x}{1 + n_n m_x/2},$$

to transform the mortality rates between the death rates and the central death rate used in these methods. In addition, since we are interested in comparing the performance of these four methods on quinquennial age groups, only the mortality rates of the groups 60-64, 65-69, 70-74, 75-79 and 80-84 are considered in this study.

Some modifications are made in applying those four methods. For the ratio method, a time series model, in addition to regression model (i.e., Equation (3)), is used in developing the model for the base group (ages 60-64). Also, since SVD cannot be applied to analyze incomplete data, we use the approximation method to estimate parameters in the Lee-Carter method. Finally, the software MORTPAK-LITE is used to solve for the eight parameters in the Heligman-Pollard method.

We use the following two error criteria to evaluate the performance of these methods:

1. MAPE (Mean Absolute Percentage Error)

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\frac{|\varepsilon_i|}{Y_i}\times 100.$$

2. RMSPE (Root Mean Square Percentage Error)

$$RMSPE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{\varepsilon_i}{Y_i}\right)^2}\times 100.$$

Here Y_i is the ith observed values (i.e. the observed mortality rates), $\varepsilon_i = Y_i - \hat{Y}_i$, where $\hat{Y}_i$ is the predicted value of Y_i.

We first obtain the predicted values for the mortality rates for years 1993 to 1997; and then we calculate the MAPE and RMSPE under each method. Figures 2.1 and 2.2 (Figure 3.1 and 3.2) are the values of MAPE and RMSPE, respectively, among four methods for the male (female).
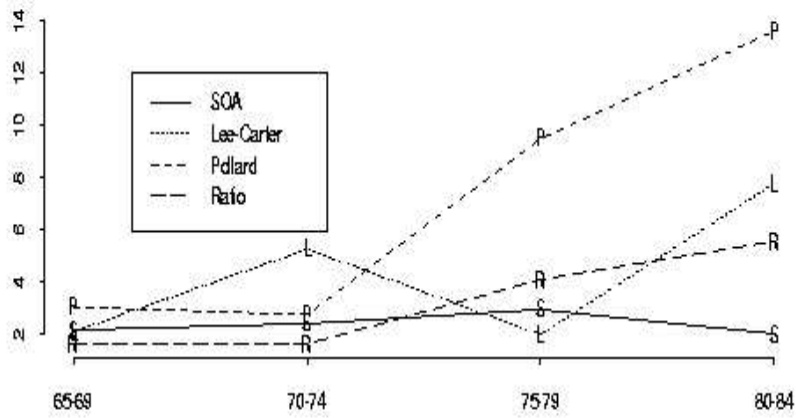
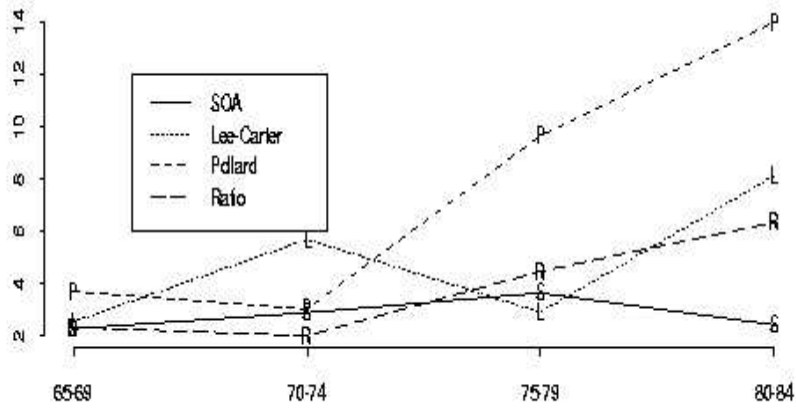**Figure 2.1 MAPE of The Mortality Rates for The Male**

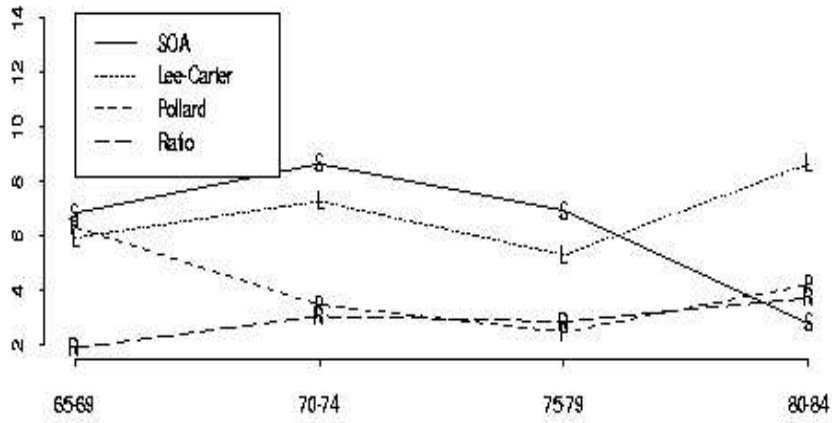**Figure 2.2 RMSPE of The Mortality Rates for The Male**

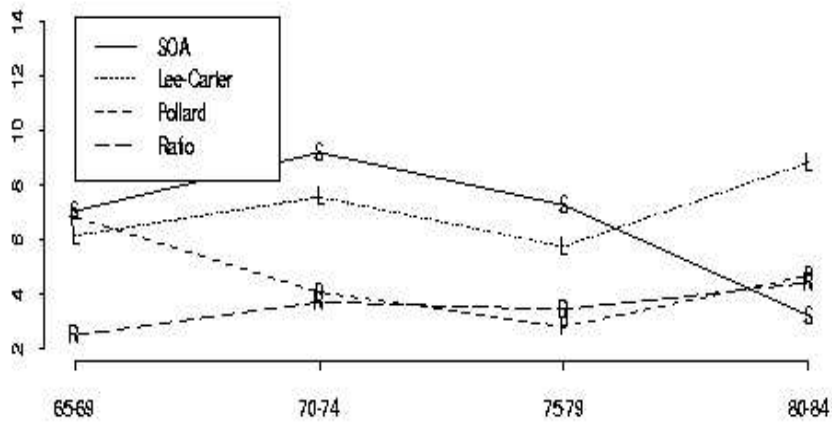**Figure 3.1 MAPE of The Mortality Rates for The Female**

Figure 3.2 RMSPE of The Mortality Rates for The Female

First, we can see that the prediction errors based on the ratio method are very stable, in relatively terms. The MAPE and RMSPE values of ratio method are ranged from 1.58 to 6.34 (refer to Appendix 1 for the details of the values) for both the male and the female. The figures also show that the prediction errors increase as the age group of interest is away from the base group (i.e., 60-64). Recall that the idea of the ratio method is to utilize the correlations of the mortality of the base group and age groups of the elderly. That is, the higher the correlations are, the smaller the prediction errors will be. Therefore, because the correlations of the higher age groups and the base group are smaller, the corresponding prediction errors (MAPE and RMSPE) are larger, which matches to the result of simulation in the previous section.

Secondly, we find that the MAPE and RMSPE values of the ratio methods are relatively smaller, compared with the other three methods. In fact, the ratio method is the best in terms of the MAPE and RMSPE criteria and based on Taiwan data in predicting the mortality rates of the groups 65-69 and 70-74 for both male and female populations, and the second best in the groups 75-79 and 80-84. This result further confirms the advantage of using a base group mortality rate to predict the mortality rates of age groups, which are highly correlated to the base group.

Thirdly, the performances of the existing methods are mixed. For example, although the SOA method is always the best in predicting the 80-84 group, its errors are the largest in the predicting the female mortality rates for other three age groups. Overall speaking, the SOA method performs better than other methods in the male mortality, while it performs the worst in the female mortality. The Heligman-Pollard method is the worst in predicting the male mortality rates, especially in higher age groups. Finally, although Lee-Carter method is recommended to predict mortality rates in previous studies, it performs not so well based on the Taiwan mortality data. A possible cause is the incomplete data, because the advantage of SVD cannot be adopted.

V. CONCLUSION

In this study, the ratio method is constructed and applied to fit Taiwan elderly mortality data. We have demonstrated the advantage of the ratio method in terms of its straightforward notion and well-performed prediction. In particular, while the incomplete data have reduced the prediction power of some other methods, the ratio method can still work well. Actually,

the idea of the ratio method is to account for the correlations between the mortality rates of certain age groups and the base group at different time point. These correlations exist because the population of a country (or a certain area) is facing similar living conditions (nutrition, environment, medical technology, and so on). We use the ratio relationship between each age group and the base group to predict the mortality rates of that age group. The main point is to choose an age group containing the most complete historical data to be the base group, so that we could have a more accurate prediction of the mortality rates.

Our simulation results demonstrate that as the incompletion of data is more severe, the advantage of using the ratio method is greater. On the other hand, the simulation results also suggest that the smaller the correlations of the mortality rates between age groups and the base group, the worse the prediction will be. One way to improve the method is to allow the base group to be varied such that the best base group (i.e., its mortality rates are highly correlated with that of the age group of interest), can be employed. Most of the time, the best base group is the nearest younger group to the age group of interest.

Since the current version of the ration method assumes the future will be in some sense like the past, just like all purely extrapolating forecasts, which may limit the precision of prediction. A possible extension of this method is to allow the time series model of predicting ratios include additional information, such as the per capita medical expenditure, per capita spending on the research related to prolong the human lives, living standard, and so on. We could use the fitted model to simulate the potential change in the mortality rates if the future environment has deviated from the past.

Appendix 1

MAPE and RMSPE of the Mortality Rates for Males and Females

Sex	Method	Error Type	Age Group				
			60-64	65-69	70-74	75-79	80-84
Male	SOA	MAPE	10.27	2.14	2.39	2.92	1.98
		RMSPE	11.01	2.28	2.9	3.65	2.44
	Lee-Carter	MAPE	9.93	2.07	5.24	1.97	7.76
		RMSPE	10.39	2.56	5.69	2.91	8.11
	Pollard	MAPE	3.12	3.03	2.73	9.49	13.61
		RMSPE	3.7	3.69	3.04	9.64	13.97
	Ratio	MAPE	10.24	1.58	1.6	4.08	5.54
		RMSPE	10.9	2.32	2.02	4.48	6.34
Female	SOA	MAPE	1.81	6.85	8.66	6.95	2.79
		RMSPE	2.18	7.05	9.19	7.26	3.22
	Lee-Carter	MAPE	1.98	5.95	7.3	5.31	8.67
		RMSPE	1.98	6.14	7.57	5.75	8.83
	Pollard	MAPE	5.89	6.33	3.48	2.49	4.22
		RMSPE	6.41	6.84	4.06	2.8	4.68
	Ratio	MAPE	1.75	1.9	3.08	2.87	3.72
		RMSPE	1.95	2.49	3.7	3.44	4.42

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